Syllabus

- General Information:

Instructor: Ying-Jen Yang (楊穎任); yangyj@uw.edu; Personal Webpage Teaching Assistant (TA): Matthew Farkas; mfarkas@uw.edu Course Website: Canvas Lectures: MWF 1:10 - 2:10 at DEN 258 Office Hour: T/Th 2:30 - 3:30 at the conference room in GRB B54 (Tentatively) TA's Office Hour: Fri 9:50-11:50 at DEM 024 (Tentatively)

- Course Description

This course serves as the first course in differential equations after learning Calculus from MATH 124 and 125. In this course, we will learn how to solve Ordinary Differential Equations (ODEs) with examples drawn from physical, chemical, biological sciences and engineering. We motivate most of our theoretical discussions from applications and aim to illustrate the logic hidden behind different solution methods. In other words, tricks are not just tricks!

- Prerequisites

Proficiency in manipulation of algebraic equations, evaluation of limits, methods of differentiation & integration at the level of MATH 124 & 125 are assumed. We will introduce and use quite a lot of linear algebra, some complex variable, and power series that are relevant to us. Exposure to linear algebra, complex variable, and Taylor series would definitely be helpful.

- Textbook and Reference

No textbook is required. A self-contained typed-up lecture note would be provided. For textbook references, the following two books are recommended:

- W.E. Boyce, R.C. DiPrima, and D.B. Meade, Elementary Differential Equations and Boundary Value Problems. 11th ed. (any edition after 9th will do. B&D, Chapter 1 ~ 5, 7 ~ 9). It is the standard textbook for undergraduate ODE class. Most of this course would be focusing on the material in it.
- 2. Steven H. Strogatz, Nonlinear Dynamics and Chaos, 2nd ed. (S, Chapter 1, 2, 5, and 6). It is an awesome textbook that is used in almost all undergraduate Dynamical Systems class around the world! We will only touch a little bit of it. You can take Amath 402/502 for a thorough introduction of the book and the subject.

- Homework Policy

Homework is assigned weekly. Please submit your scanned homework to Canvas. No late homework is allowed since the solution of it will be available right after the deadline. At the end of the quarter, the homework with the lowest grade will be dropped.

You have to show all your work in a neat and organized way with answer boxed. You are highly encouraged to use IAT_EX to type up your work. I'll recommend the LyX interface, which is a "Word-version" of IAT_EX that allows you to see equations and figures when editing. Handwritten homework is also allowed. You will need to scan your handwritten work as a letter-size, portrait PDF file. Collaboration is encouraged. But your homework should be written in your own words. If your submitted work is hard to read for the grader or is a copy to others', the grader reserves the right to give you a zero.

Homework would be graded rigorously by the TA. Some problems have no partial credit. They are typically problems like "solve this equation" that you can easily check your answer by plugging in your answer to the equation and/or with your classmates. This policy forces you to get used to check your answer, which is a good habit in exam or in research. In those no partial credit problems, you still have to show all your work and box your answer to get full credit.

- Exams and Grade

We will have two midterms and one final exam, all 60 minutes in-class:

Midterm 1: Jul 17 (Wed), Chap $1 \sim 2$

Midterm 2: Aug 07 (Wed), Chap $1 \sim 4$

Final Exam: Aug 23 (Fri), Chap $1 \sim 5$

All examinations are accumulative. Midterm 2 will focus on new material. In all the exams, each student is allowed to have **one hand-written**, **letter-size**, **two-sided allowed**, **note sheet**. No calculator is allowed. If you find a "really ugly" answer in the exam, there is a high chance you get the answer wrong (extra clue to check your work, how nice, right?)

The course grade in percentage would be computed by 50% Homework, 15% Higher Midterm, 10% Lower Midterm, 25% Final. The conversion from percentage to GPA 4.0 scale will be determined at the end of the quarter. I may refer to historic record to determine the conversion formula. Historically, the median is about 3.5 and about 5-10% of the students get 4.0.

- Academic Responsibility

Students shall abide by the University of Washington Academic Responsibility policies. Violations will be reported to the appropriate Dean's Representative and through the web-page for Community Standards and Student Conduct. The instructor reserves the right to assign a failing grade for the course for serious violations of student conduct.

Note: Use of websites or online forums which provide solutions for class assignments is not allowed. You are also not allowed to distribute course materials to any individual or corporation outside of this course without the instructor's consent.

Collaboration and study groups are highly encouraged! Copying or submitting work that is identical to a classmate's work or online solution is academic misconduct and will be reported according to the policies communicated by Community Standards & Student Conduct (CSSC). Any form of dishonesty in an assignment will lead to a zero on the assignment. Other consequences, including a failing grade in the course, will be determined based on the seriousness of the offense or multiple offenses at the instructor's discretion.

- Access and Accommodations:

If you have already established accommodations with Disability Resources for Students (DRS), please communicate your approved accommodations to me at your earliest convenience so we can discuss what we will do for this course. If you have not yet established services through DRS, but would like to have one, you are welcome to contact DRS at 206-543-8924 or uwdrs@uw.edu. DRS offers resources and coordinates reasonable accommodations for students with disabilities and/or temporary health conditions. Reasonable accommodations are established through an interactive process between you, your instructor(s) and DRS. It is the policy and practice of the University of Washington to create inclusive and accessible learning environments consistent with federal and state law.

- Outline

Chap 1: Differential Equations and Dynamical System

An overall introduction on the topic. Get familiar with the terminology and know how to classify different problems so that we can know how to solve them.

Reference: <u>S Chap 1, 2</u>; <u>B&D Chap 1.2, 1.3</u>

- 1.1 Introduction: Importance of DE and DS
- 1.2 Classification of ODE with examples from physics, chemistry, and biology
- 1.3 1D Dynamical System and Nonlinearity

Chap 2: 1st Order ODE

Given a 1st order ODE, how to find a suitable solution method and solve it.

- Reference: B&D Chap 2
 - 2.1 Separable ODE
 - 2.2 Linear: integration factor
 - 2.3 Substitution: Bernoulli and "Homogeneous"
 - 2.4 Exact 1st order ODE
 - 2.5 Integration Factors to make an ODE exact

Chap 3: General Theory for linear ODEs

Knowing how to solve 1st order 1-D ODE, extend it to solve linear ODEs with higher orders and/or higher dimensions. Reference: B&D Chap 7

- 3.1 Concept and the General Solution
- 3.2 Matrix and Determinant
- 3.2 Eigenvalues, Eigenvectors, and Diagonalization
- 3.3 Solution to System of linear ODEs and Wronskian

Chap 4: 2-D autonomous, linear ODE

Apply and extend Chap. 3 to solve 2-D/2nd order, autonomous, linear ODEs extensively. Reference: B&D Chap 3

4.1 Homogeneous: distinct real eigenvalues

- 4.2 Homogeneous: complex conjugated eigenvalues
- 4.3 Homogeneous: repeated eigenvalues and reduction of order
- 4.4 Summary for Homogeneous cases and Fixed point classification
- 4.5 Inhomogeneous cases

Chap 5: 2nd order, non-autonomous, linear ODE

Learn how to solve or find local power series solutions to 2nd order, non-autonomous, linear ODE. Reference: B&D Chap 5

- 5.1 Special cases with closed-form solutions: Cauchy-Euler, and Exact
- 5.2 Taylor series and Convergence
- 5.3 Ordinary point and Power Series solution

Chap 6: A short glimpse of Nonlinear Dynamics

Have a taste on how to analyze nonlinear ODEs with our theories for linear systems. Reference: B&D Chap 2.7, 9; S Chap 5,6,7

6.1 Nonlinear ODEs and a geometrical way of thinking

6.2 Linear stability analysis

Standard undergrad-level ODE Topics not covered:

- Laplace Transform: (will definitely be covered in graduate-level complex analysis) an efficient way to solve inhomogeneous, autonomous, linear ODE (Reference: B&D Chap 6)
- Frobenius Method: (will definitely be covered in graduate-level ODE course) series method for regular singular point of non-autonomous, linear ODE (Reference: B&D Chap 5)

Courses Tangential to this course

Amath 301 (Numerical Methods): how to solve many different math problems via basic computer programming

Amath 352 (Linear Algebra): more about matrices, eigenvalue problems, and application to numerical methods

Amath 353 (Partial Differential Equations): how to solve equations with partial derivatives in them!

Amath 383 (Mathematical Modeling): a research-like course for how to apply math to solve real world problems!

Amath 401 (Complex Variables): how to take your Calculus to the next level in a applied sense.

Amath 402 (Dynamical Systems): where you analyze nonlinear ODEs and read the awesome textbook by Strogatz