AMATH 353 Class Content Spring 2019

Quiz content

We will do mastery based grading for the quizzes. Each quiz will be approximately 30 minutes and will take place at the beginning of class. Quizzes will be graded with very little to no partial credit. If you are not pleased with your grade, you may retake the quiz at a later date. Upon choosing to retake the quiz, you must agree to forfeit your old grade and accept the new grade you get on the quiz. Each retake may be (not significantly) longer than 30 minutes, depending on difficulty. There will be no final for this course. The last retake opportunity will replace the final and will take place on June 10, 2019. Any quiz may be retaken on this date.

The quizzes will cover the following topics.

• Quiz 1 - basic understanding of PDEs.

Students will be able to...

- give a geometric and physical interpretation of a PDE solution, without finding the solution;
- algebraically check whether a function is a solution of a PDE;
- classify a PDE in terms of its order, as homogeneous or non homogeneous, and as linear or nonlinear;
- recognize that linear and homogeneous PDEs have the superposition property and nonlinear PDEs do not;
- recognize different side conditions as initial or boundary conditions;
- recognize the boundary conditions of a PDE as periodic, decaying, Dirichlet, or Neumann.
- determine the effect of each term in a PDE on the solution; and
- change the coordinates in a given problem to rewrite the PDE or side conditions.

• Quiz 2 - Wave solutions of PDEs

Students will be able to...

- find **all** traveling wave solutions of a given PDE;
- linearize a nonlinear PDE about the trivial solution;
- find the dispersion relation for a linear or nonlinear PDE;
- use the dispersion relation to qualitatively describe solutions of a PDE; and
- find solutions of the wave equation on the whole or half-line using d'Alembert's solution.
- Quiz 3 Boundary Value Problems and Fourier Series Students will be able to...

- use separation of variables to write one linear PDE as a collection of ODEs;
- use separation of variables to find standing waves of the wave equation;
- solve the wave equation using superposition of standing waves;
- find the Fourier series of a general function; and
- solve second-order linear PDEs using Fourier series.
- Quiz 4 Conservation law PDEs and the method of characteristics

Students will be able to...

- convert a conservation law to a PDE;
- obtain solutions of homogeneous and nonhomogeneous general linear PDEs via the method of characteristics; and
- obtain solutions of nonlinear PDEs via the method of characteristics, up until the breaking time.
- Quiz 5 Shock and rarefaction waves

Students will be able to...

- compute the breaking time for a nonlinear PDE solution;
- find the Rankine-Hugoniot condition for discontinuous solutions;
- use the Rankine-Hugoniot condition to find shock wave solutions for nonlinear PDEs; and
- find rarefaction wave solutions for nonlinear PDEs.

Applications, deeper theory, and side topics

There will be other topics that show up in the course related to the above critical topics. Because they are less fundamental, more difficult to grasp, or out of the scope of the course, they will not be on quizzes directly. They may, however, appear on homework. They are all interesting topics, and I can guide interested individuals to more resources if necessary. These include:

- wave trains;
- phase and group velocity and the difference between them;
- deriving PDE models from physical laws;
- convergence properties of Fourier Series;
- multidimensional problems / eigenfunctions of differential operators;
- fourier transforms and how to solve differential equations using them;
- traffic flow problems and modeling of them;
- the viscosity method for hyperbolic systems;
- rarefaction and shock waves combined; and
- weak solutions/function spaces;

In particular, we will discuss Fourier transforms, weak solutions and function spaces, or the viscosity method depending on the interest of the class, if and only if time permits.