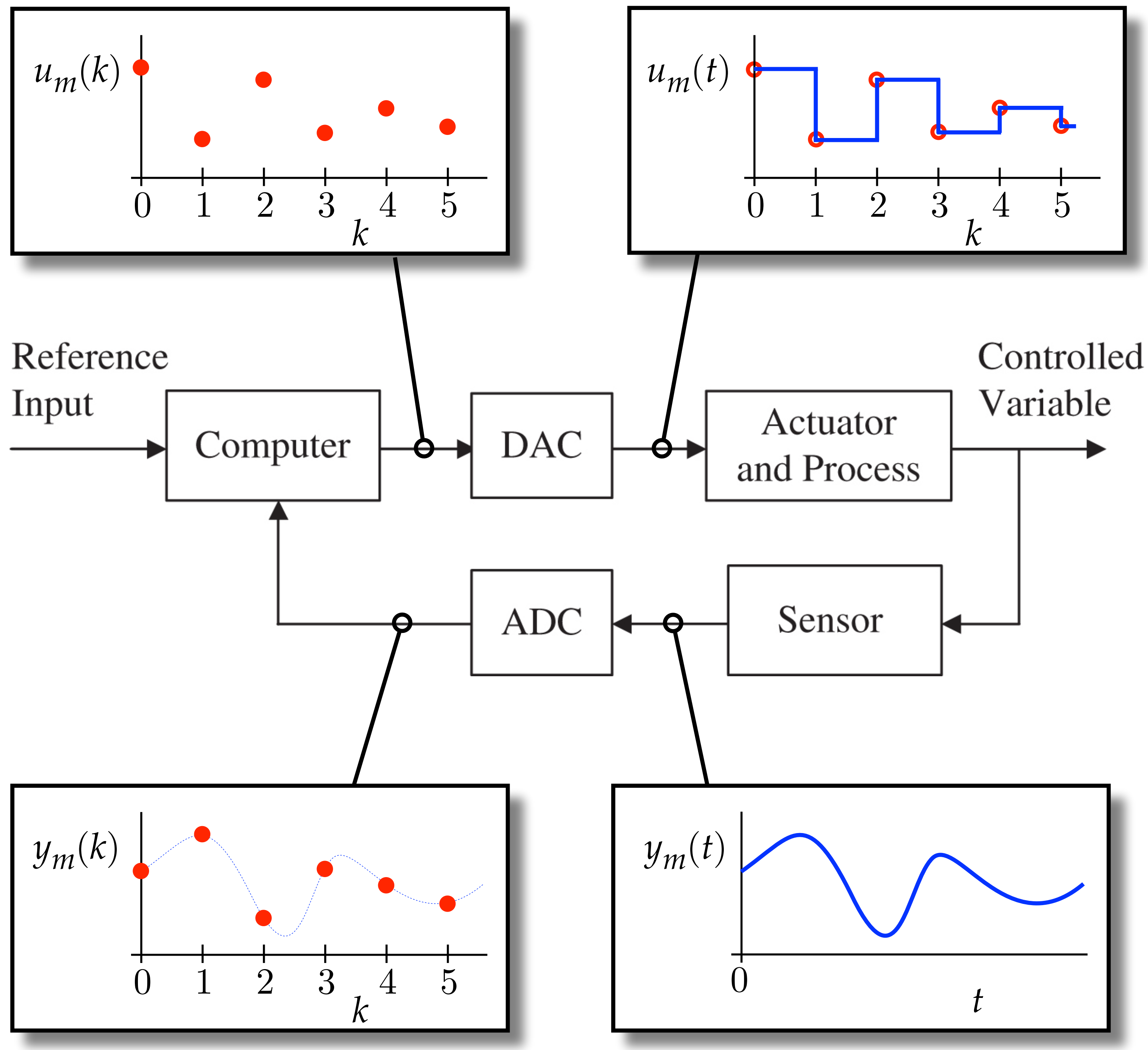
Digital to analog converters (DAC), analog to digital converters (ADC)

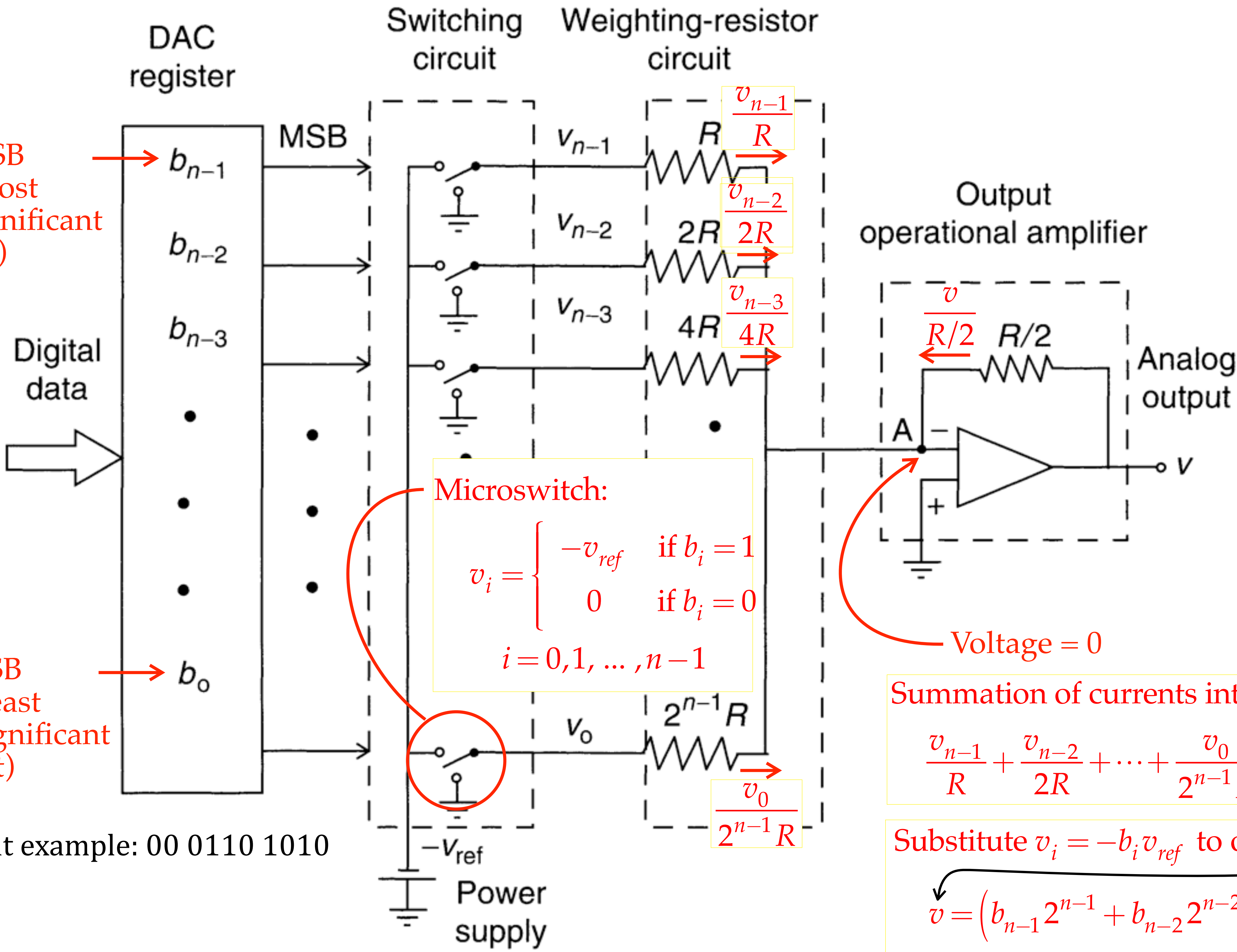


$y_m(k) \triangleq y_m(k\,\Delta T),\ k = 0, 1, 2, \ldots$

- Resolution: given in number of bits (e.g. 10 or 16 bits, corresponding to dividing the voltage range into 2^10=1024 or 2^16=65536 intervals)

- speed (bandwidth): minimum interval ΔT

Digital to Analog Conversion



MSB (most significant bit)

LSB (least significant bit)

10-bit example: 00 0110 1010

Microswitch:

$$v_i = \begin{cases} -v_{ref} & \text{if } b_i = 1 \\ 0 & \text{if } b_i = 0 \end{cases}$$

$$i = 0, 1, \ldots, n-1$$

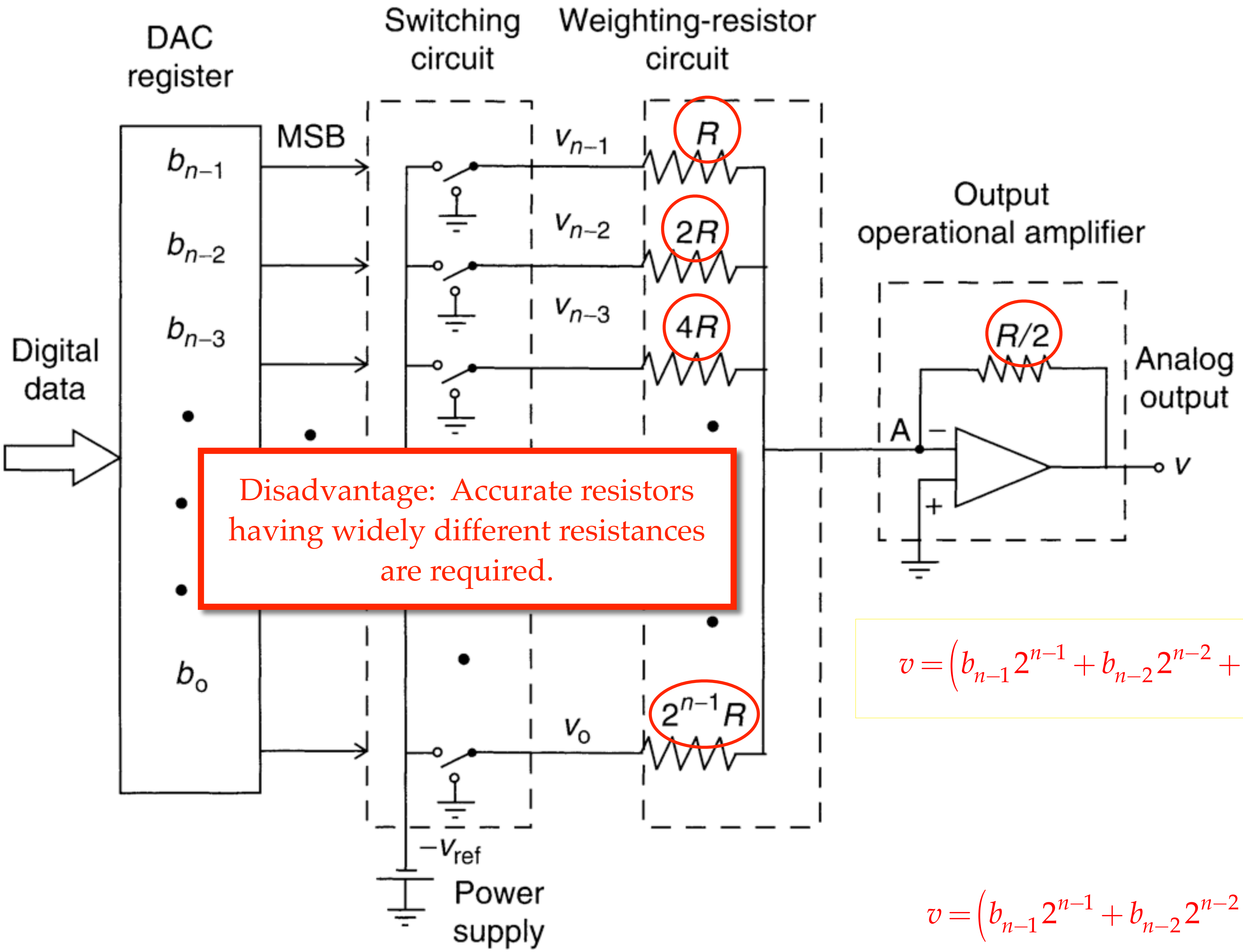
Summation of currents into node A:

$$\frac{v_{n-1}}{R} + \frac{v_{n-2}}{2R} + \cdots + \frac{v_0}{2^{n-1}R} + \frac{v}{R/2} = 0$$

Substitute $v_i = -b_i v_{ref}$ to obtain

$$v = \left(b_{n-1}2^{n-1} + b_{n-2}2^{n-2} + \cdots + b_0 2^0\right)\frac{v_{ref}}{2^n}$$

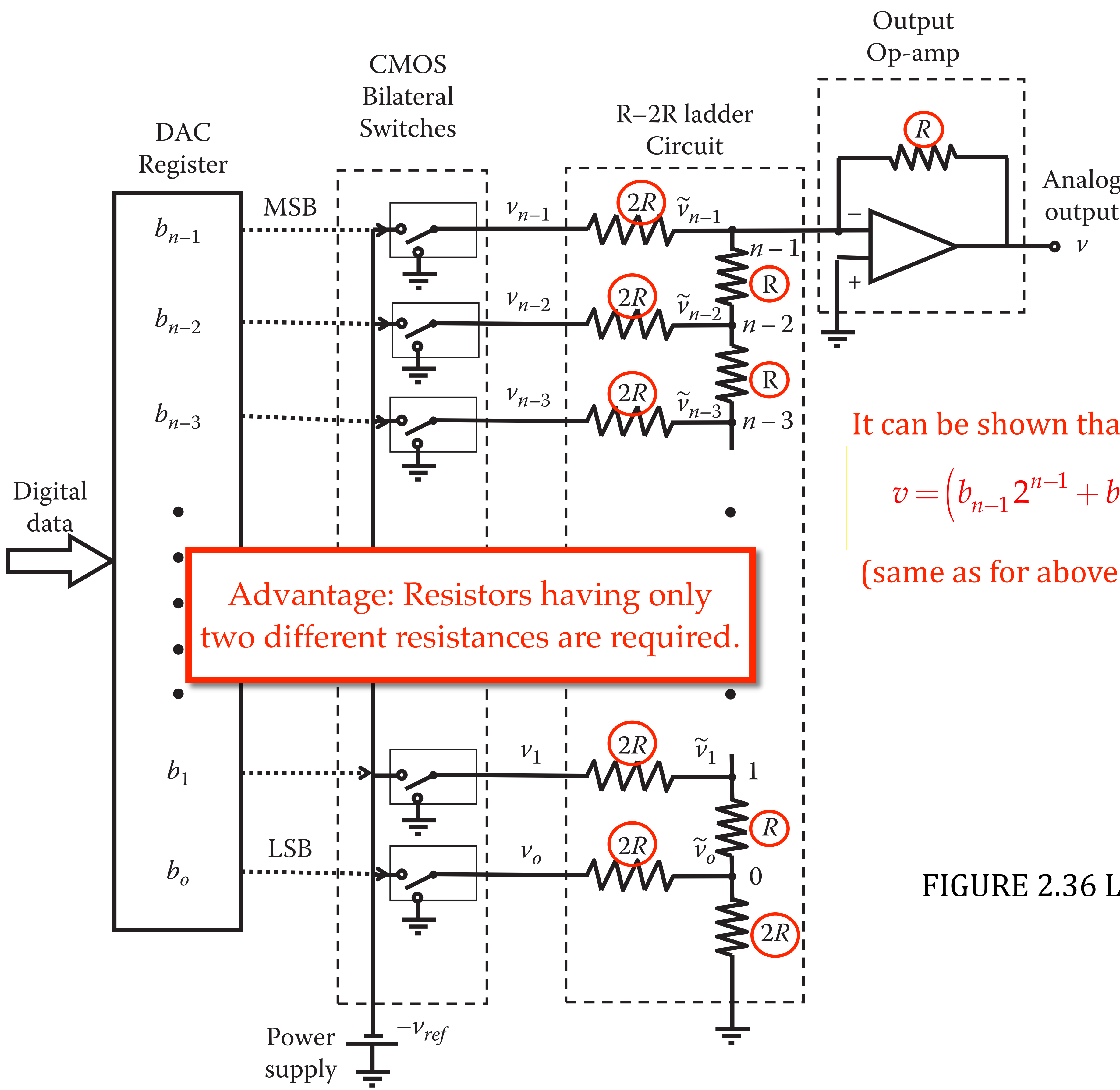
Digital to Analog Conversion



Disadvantage: Accurate resistors having widely different resistances are required.

$$v = \left(b_{n-1}2^{n-1} + b_{n-2}2^{n-2} + \cdots + b_0 2^0\right)\frac{v_{ref}}{2^n}$$

$$v = \left(b_{n-1}2^{n-1} + b_{n-2}2^{n-2} + \cdots + b_0 2^0\right)\frac{v_{ref}}{2^n}$$



It can be shown that:

$$v = \left(b_{n-1} 2^{n-1} + b_{n-2} 2^{n-2} + \cdots + b_0 2^0 \right) \frac{v_{ref}}{2^n}$$

(same as for above "weighted resistor DAC")

FIGURE 2.36 Ladder DAC circuit.

analog to digital converters (ADC)

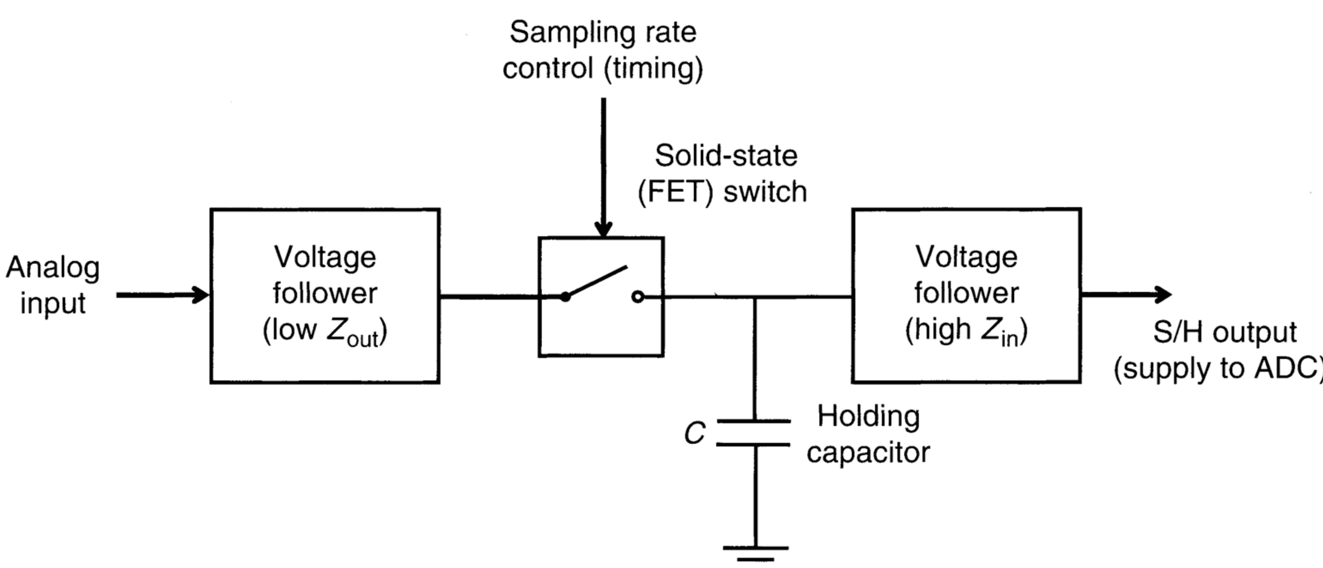


Figure 2.40 The circuit of a sample-and-hold chip.

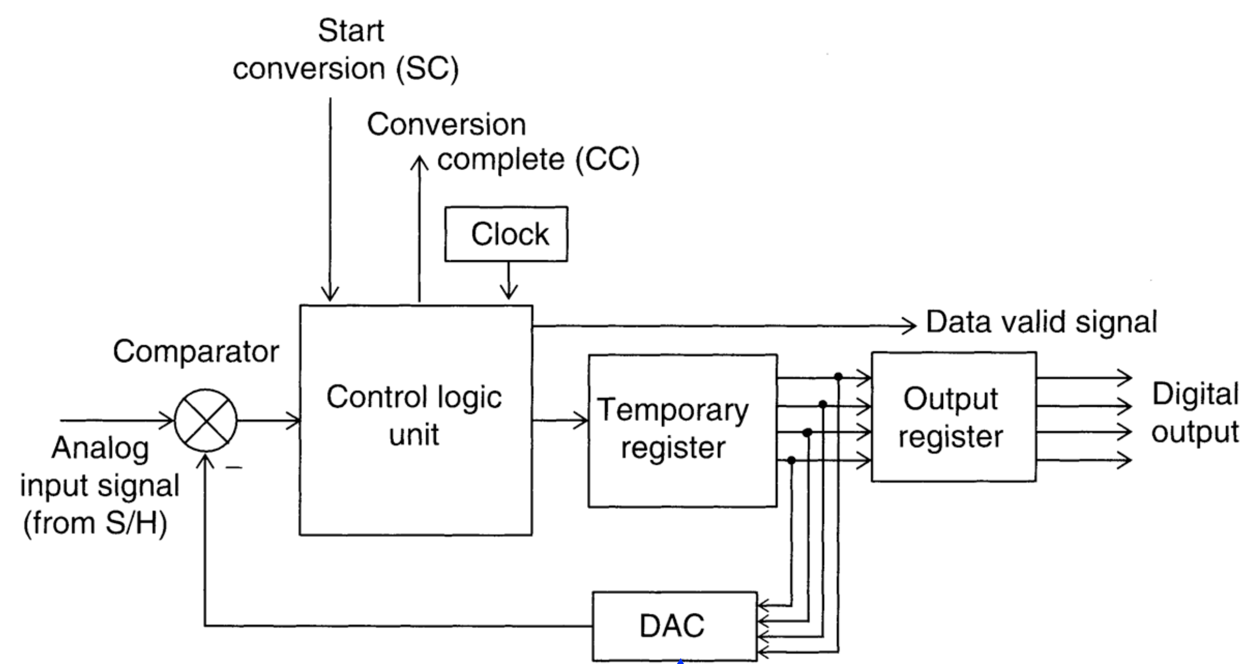


Figure 2.38 Successive approximation ADC.

For example, a weighted-resistance or ladder DAC

Example: 4-bit ADC with Signal Range 0-5v

Input Range: 0-5 v

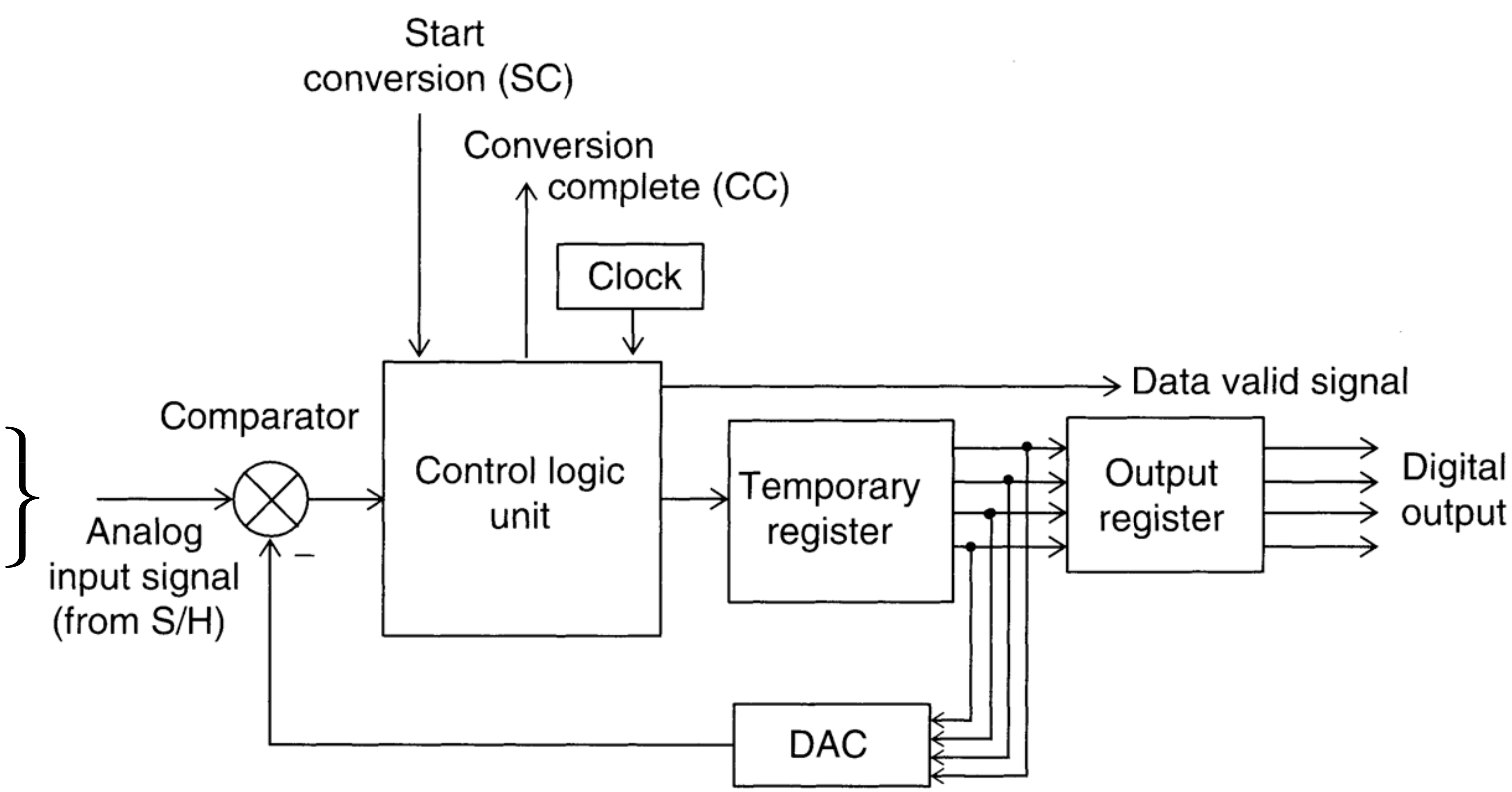


Figure 2.38 Successive approximation ADC.

Base 2		Base 10	Volts
0000	=	0	0
0001	=	1_{10}	1/3
0010	=	2_{10}	2/3
0011	=	3_{10}	1
0100	=	4_{10}	1 1/3
0101	=	5_{10}	1 2/3
0110	=	6_{10}	2
0111	=	7_{10}	2 1/3
1000	=	8_{10}	2 2/3
1001	=	9_{10}	3
1010	=	10_{10}	3 1/3
1011	=	11_{10}	3 2/3
1100	=	12_{10}	4
1101	=	13_{10}	4 1/3
1110	=	14_{10}	4 2/3
1111	=	15_{10}	5

Suppose: Analog Input $=$ 3 1/6 v

Targeted functionality:

$$0\text{ V} \leq \text{Analog Input} < 1/3\text{ V} \Rightarrow \text{Digital Output} = 0000$$

$$1/3\text{ V} \leq \text{Analog Input} < 2/3\text{ V} \Rightarrow \text{Digital Output} = 0001$$

$$2/3\text{ V} \leq \text{Analog Input} < 1\text{ V} \Rightarrow \text{Digital Output} = 0010$$

$$\vdots \qquad\qquad \vdots$$

Example: 4-bit ADC with Signal Range 0-5v

Input Range: 0-5 v }

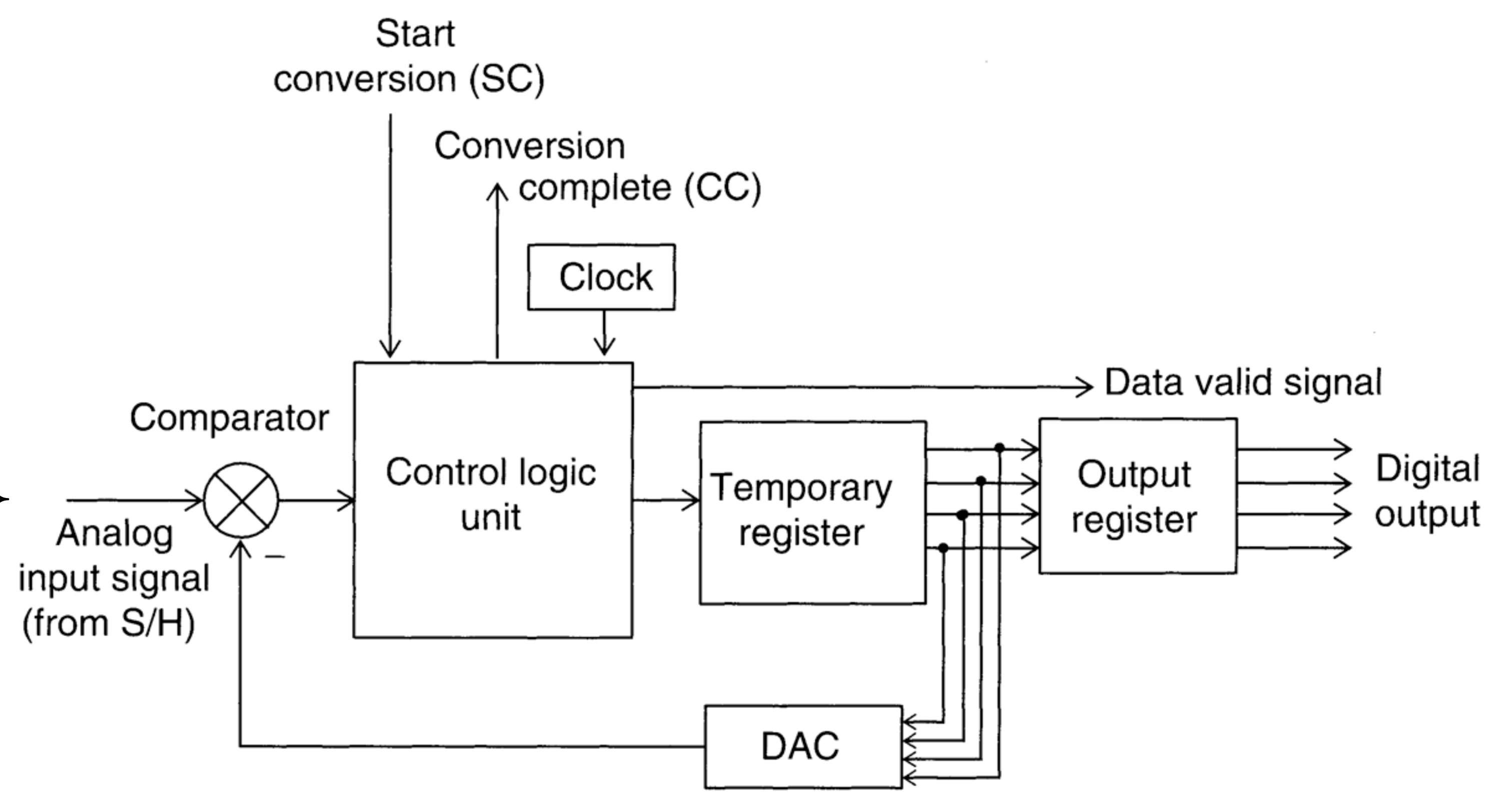


Figure 2.38 Successive approximation ADC.

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0001	=	1_{10}	1/3
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0011	=	3_{10}	1
0100	=	4_{10}	1 1/3
0101	=	5_{10}	1 2/3
0110	=	6_{10}	2
0111	=	7_{10}	2 1/3
1000	=	8_{10}	2 2/3
1001	=	9_{10}	3
1010	=	10_{10}	3 1/3
1011	=	11_{10}	3 2/3
1100	=	12_{10}	4
1101	=	13_{10}	4 1/3
1110	=	14_{10}	4 2/3
1111	=	15_{10}	5

Analog Input = 3 1/6 v

Bit 3: Temporary Register = 1000
(first guess is always 1000)

⇒ DAC Output = 2 2/3 v

⇒ Analog Input – DAC Output = Positive

⇒ Bit 3 = 1

Bit 2: Temporary Register = 1100

⇒ DAC Output = 4 v

⇒ Analog Input – DAC Output = Negative

⇒ Bit 2 = 0

Example: 4-bit ADC with Signal Range 0-5v

Input Range: 0-5 v

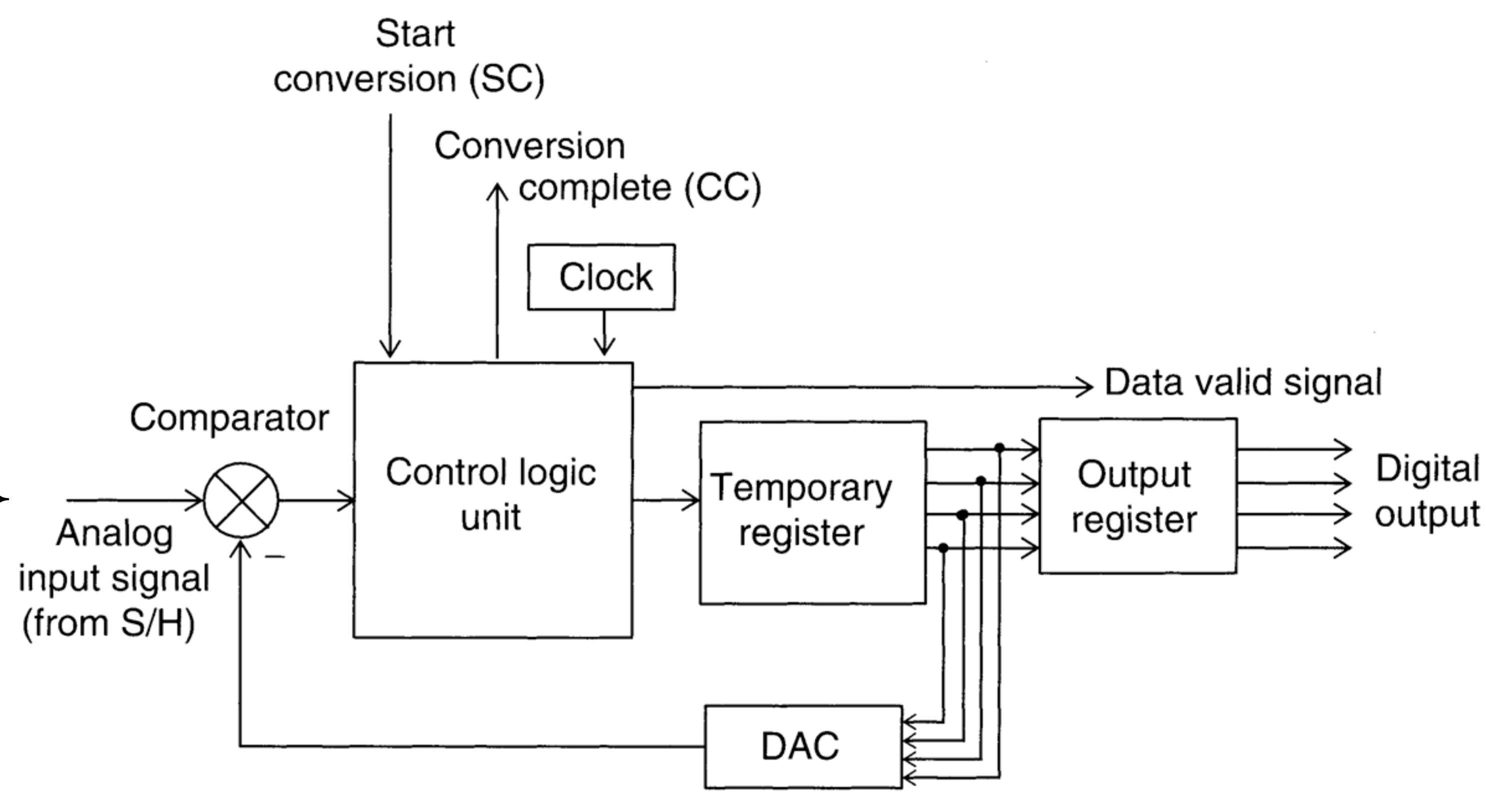

Figure 2.38 Successive approximation ADC.

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1000	=	8_{10}	2 2/3
1001	=	9_{10}	3
1010	=	10_{10}	3 1/3
1011	=	11_{10}	3 2/3
1100	=	12_{10}	4
1101	=	13_{10}	4 1/3
1110	=	14_{10}	4 2/3
1111	=	15_{10}	5

Analog Input = 3 1/6 v

Bit 1: Temporary Register = 1010

⇒ DAC Output = 3 1/3 v

⇒ Analog Input – DAC Output = Negative

⇒ Bit 1 = 0

Bit 0: Temporary Register = 1001

⇒ DAC Output = 3 v

⇒ Analog Input – DAC Output = Positive

⇒ Bit 0 = 1

Example: 4-bit ADC with Signal Range 0-5v

Input Range: 0-5 v

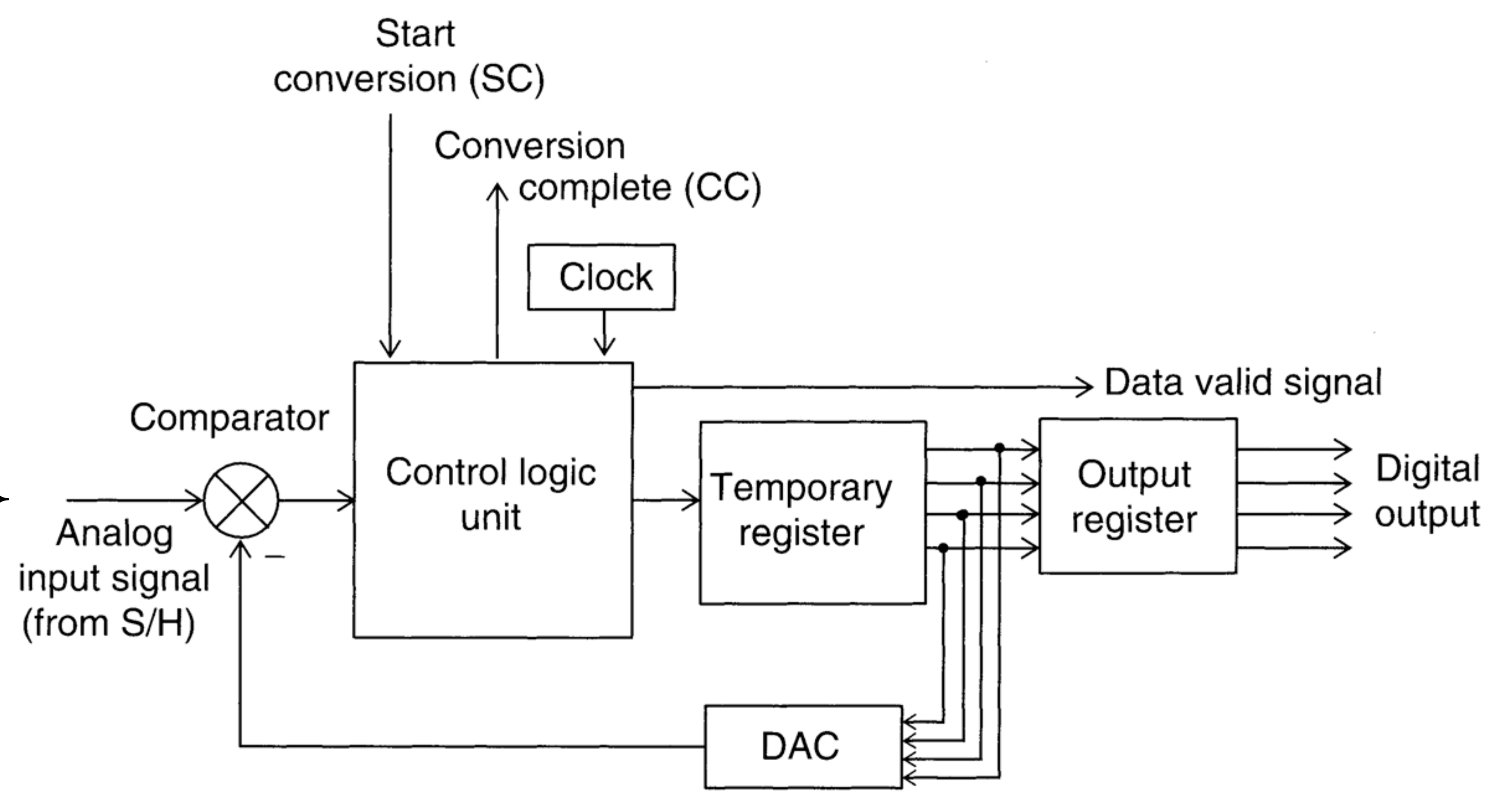

Figure 2.38 Successive approximation ADC.

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1000	=	8_{10}	2 2/3
1001	=	9_{10}	3
1010	=	10_{10}	3 1/3
1011	=	11_{10}	3 2/3
1100	=	12_{10}	4
1101	=	13_{10}	4 1/3
1110	=	14_{10}	4 2/3
1111	=	15_{10}	5

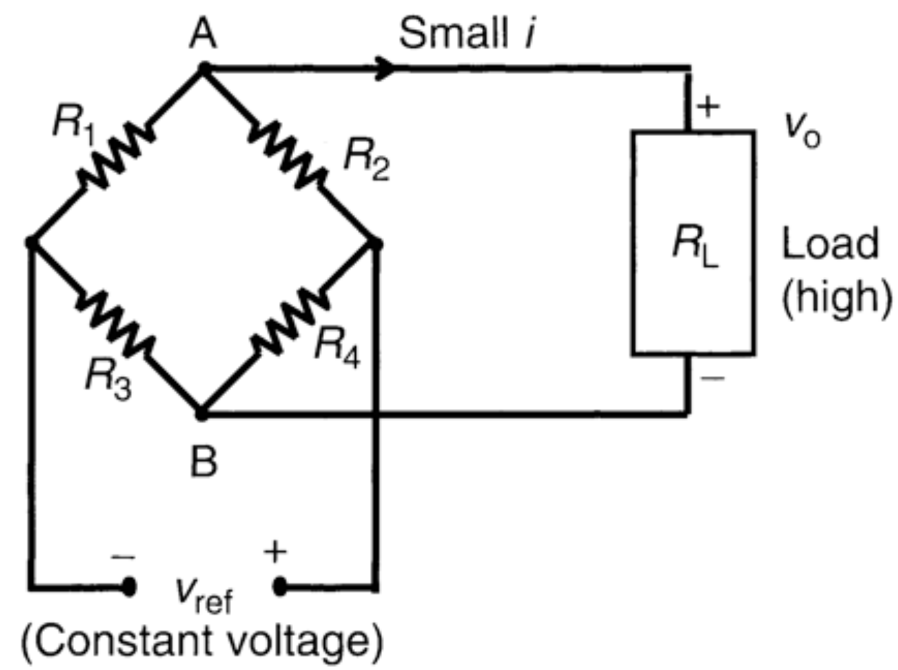
Analog Input = 3 1/6 v

Finish: Temporary Register = 1001

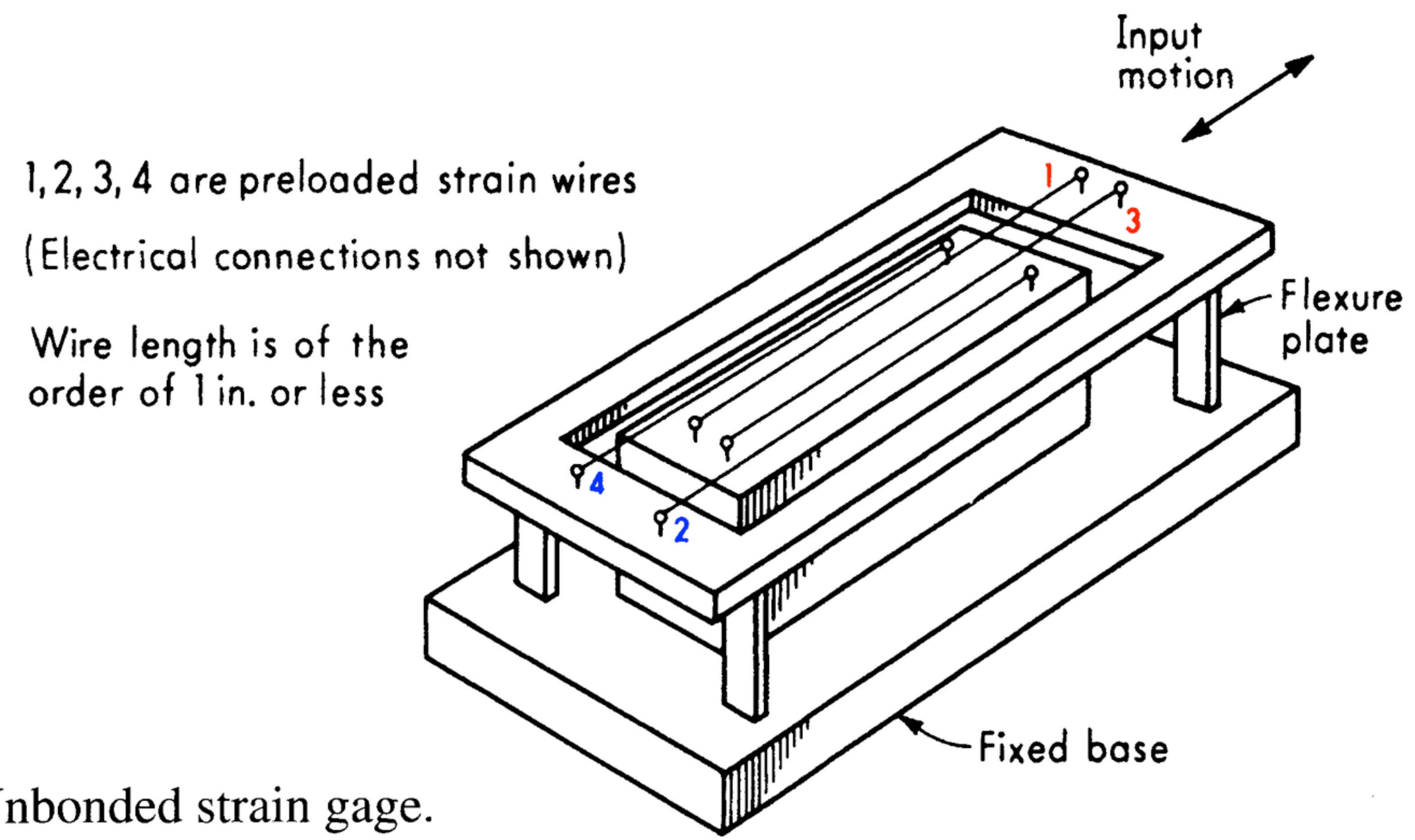
$\Rightarrow$ Output Register = 1001

Conversion Complete!

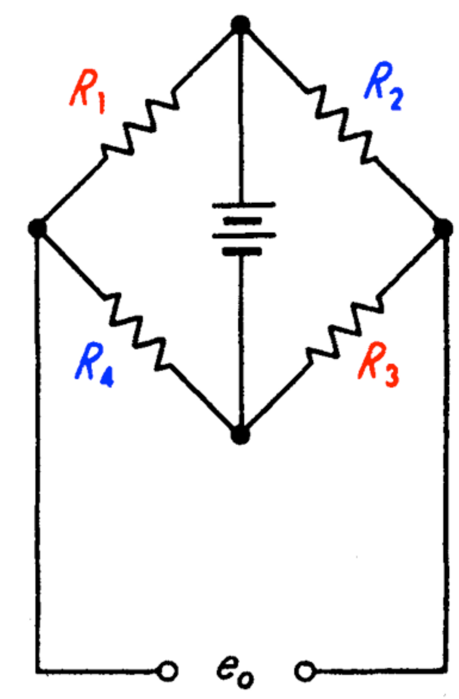
Constant-Voltage Resistance Bridge



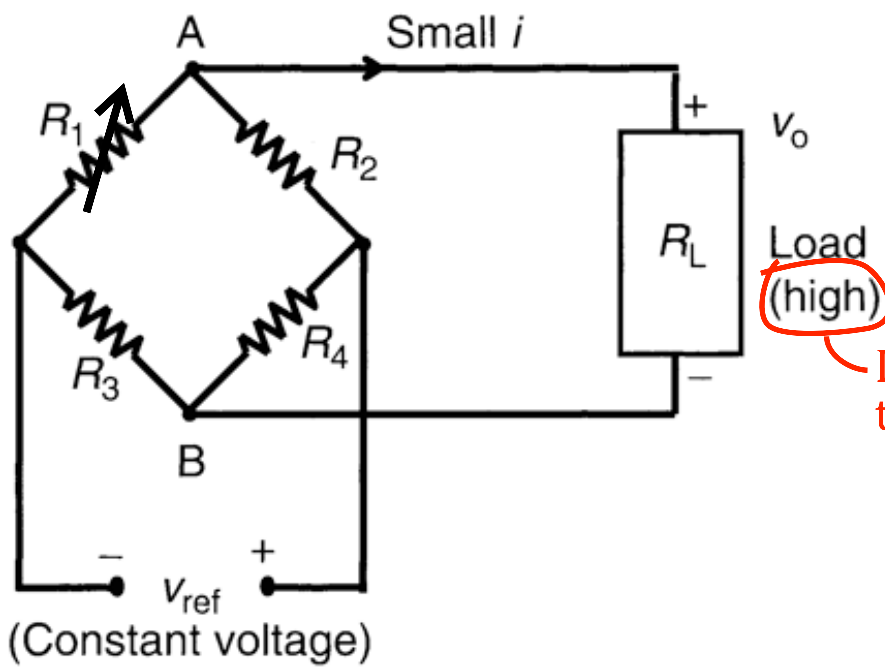
Example Application: An Unbonded Strain Gage



Unbonded strain gage.



Constant-Voltage Resistance Bridge



If $R_L >> R_i$, then negligible current flows through the load.

$$v_o = v_A - v_B = \left[\frac{R_1}{R_1+R_2} - \frac{R_3}{R_3+R_4}\right] v_{ref} = \left[\frac{R_1R_4 - R_2R_3}{(R_1+R_2)(R_3+R_4)}\right] v_{ref}$$

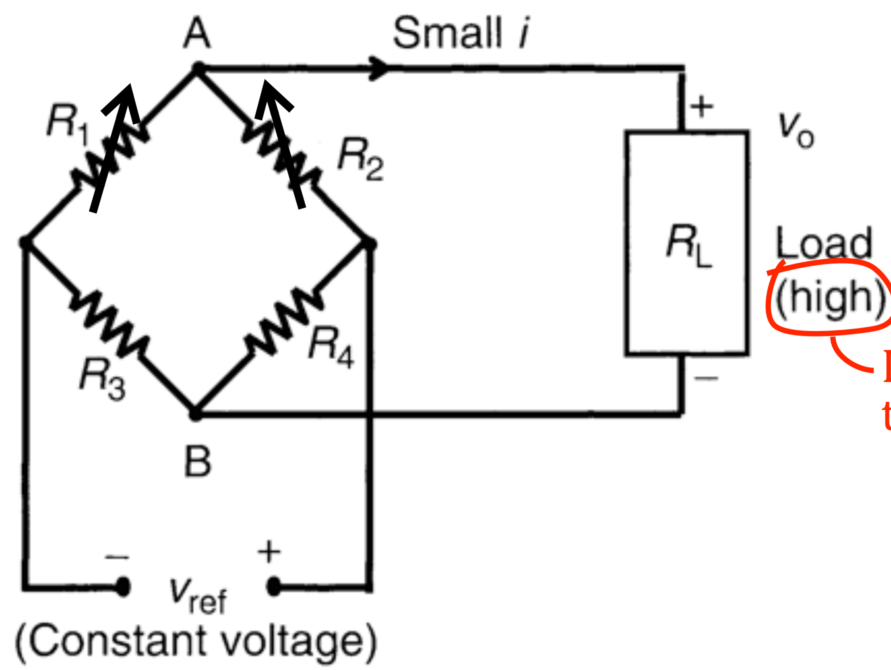
$$= 0 \text{ when } \frac{R_1}{R_2} = \frac{R_3}{R_4}$$ ← Balanced Bridge Condition

With $R_2 = R_3 = R_4 = R$ and $R_1 = R + \delta R$:

$$\delta v_o = v_o\Big|_{\substack{R_1=R+\delta R\\ R_2=R_3=R_4=R}} - v_o\Big|_{R_1=R_2=R_3=R_4=R} = \left[\frac{(R+\delta R)(R)-(R)(R)}{(R+\delta R+R)(R+R)}\right] v_{ref} = \left[\frac{R(\delta R)}{4R^2+2R(\delta R)}\right] v_{ref}$$

$$= \left(\frac{\delta R/R}{4+2\delta R/R}\right) v_{ref}$$ ← δv_o due to δR is a *nonlinear* function of δR

Constant-Voltage Resistance Bridge



If $R_L >> R_i$, then negligible current flows through the load.

$$v_o = v_A - v_B = \left[\frac{R_1}{R_1+R_2} - \frac{R_3}{R_3+R_4}\right] v_{ref} = \left[\frac{R_1R_4 - R_2R_3}{(R_1+R_2)(R_3+R_4)}\right] v_{ref}$$

$$= 0 \text{ when } \frac{R_1}{R_2} = \frac{R_3}{R_4} \longleftarrow \text{Balanced Bridge Condition}$$

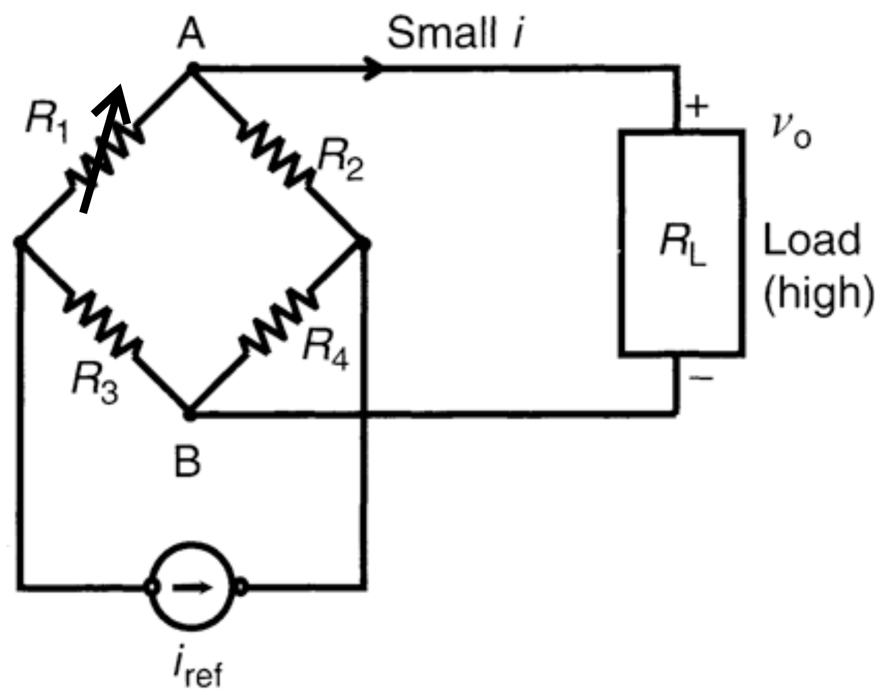
With $R_3 = R_4 = R$ and $R_1 = R + \delta R$ and $R_2 = R - \delta R$:

$$\delta v_o = v_o\Big|_{\substack{R_1=R+\delta R\\ R_2=R-\delta R\\ R_3=R_4=R}} - v_o\Big|_{R_1=R_2=R_3=R_4=R} = \left[\frac{(R+\delta R)(R)-(R-\delta R)(R)}{(R+\delta R+R-\delta R)(R+R)}\right] v_{ref} = \left[\frac{2R(\delta R)}{4R^2}\right] v_{ref}$$

$$= \left(\frac{v_{ref}}{2R}\right)\delta R \longleftarrow \delta v_o \text{ proportional to } \delta R!!$$

Example 2.11 in de Silva treats this same example and gets a wrong result!

Constant-Current Resistance Bridge



(constant current)

For constant-voltage bridge this was $\dfrac{\delta R/R}{4+2\delta R/R}$

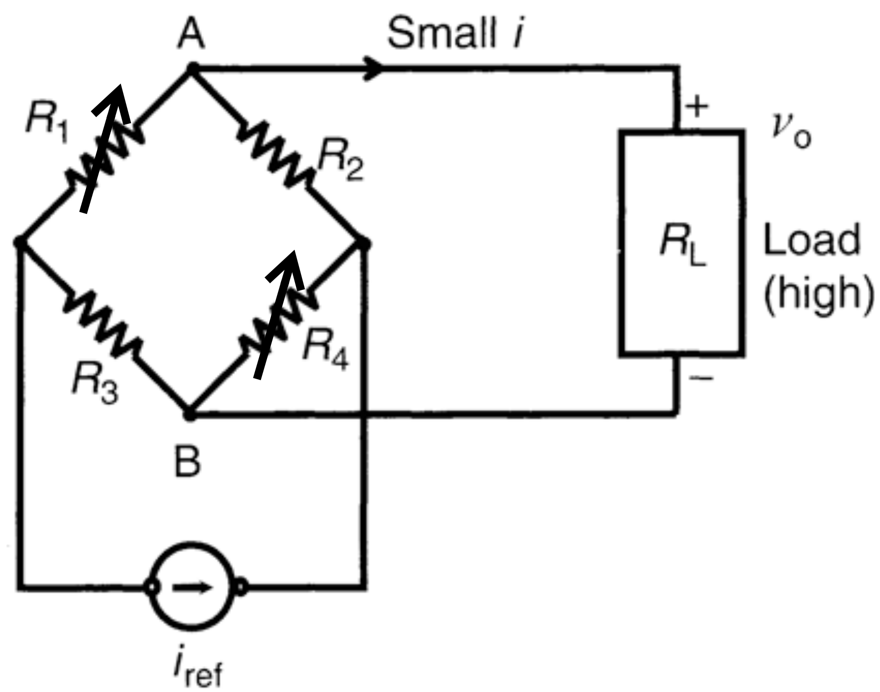
With $R_2 = R_3 = R_4 = R$ and $R_1 = R + \delta R$:

$$\delta v_o = v_o\Big|_{\substack{R_1=R+\delta R\\ R_2=R_3=R_4=R}} - v_o\Big|_{R_1=R_2=R_3=R_4=R} = \left(\frac{\delta R/R}{4+\delta R/R}\right) R\, i_{ref}$$

δv_o due to δR is a *nonlinear* function of δR

(Derivation in Sec. 2.8.2)

Constant-Current Resistance Bridge



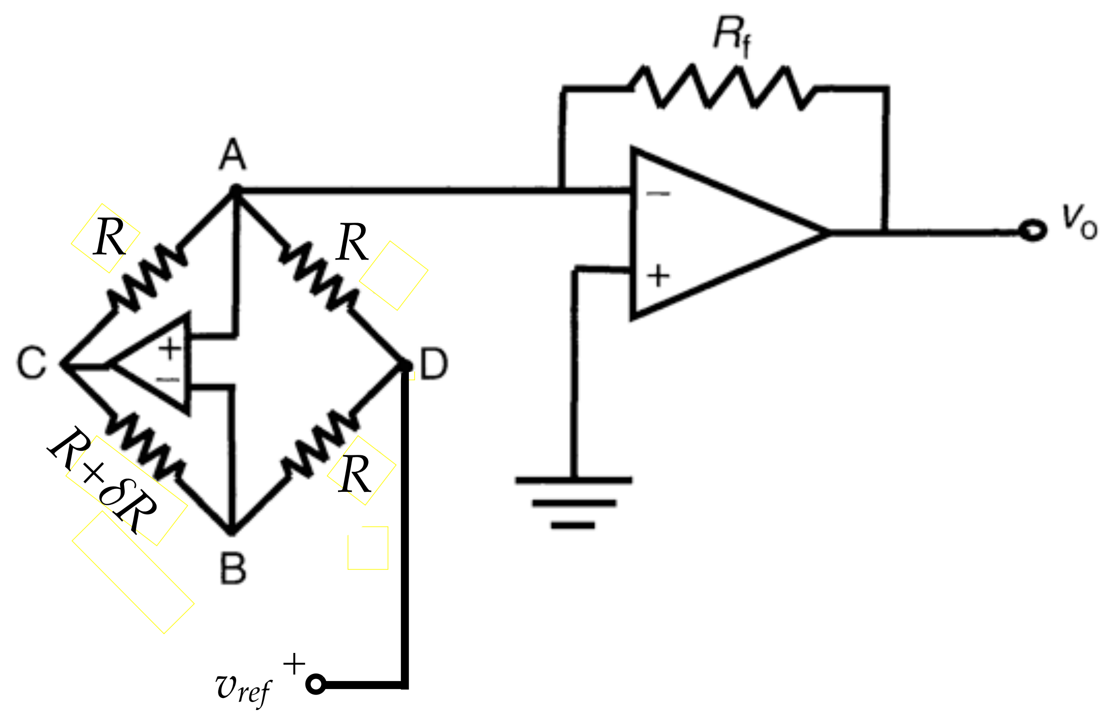
(constant current)

With $R_2 = R_3 = R$ and $R_1 = R + \delta R$ and $R_4 = R + \delta R$:

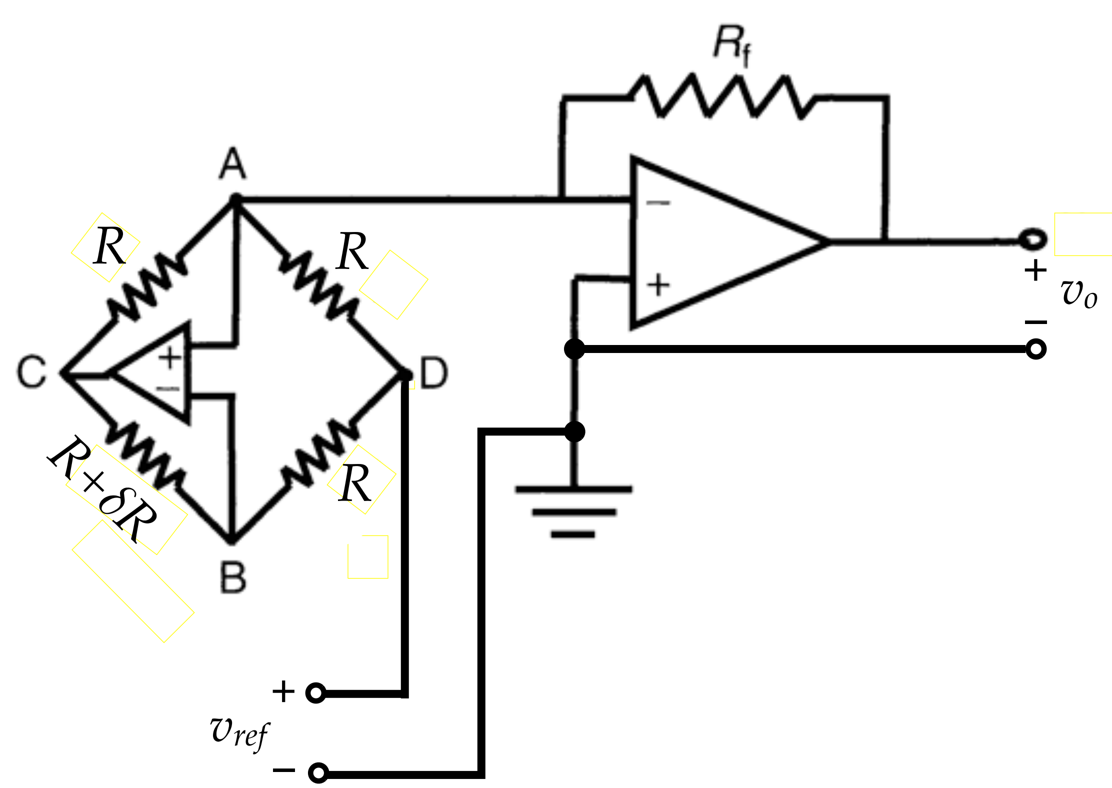
$$\delta v_o = \left. v_o \right|_{\substack{R_1 = R+\delta R \\ R_4 = R+\delta R \\ R_2 = R_3 = R}} - \left. v_o \right|_{R_1 = R_2 = R_3 = R_4 = R} = \left(\frac{\delta R / R}{2} \right) R\, i_{ref} \quad \longleftarrow \quad \delta v_o \text{ proportional to } \delta R!!$$

(Derivation in Example 2.12)

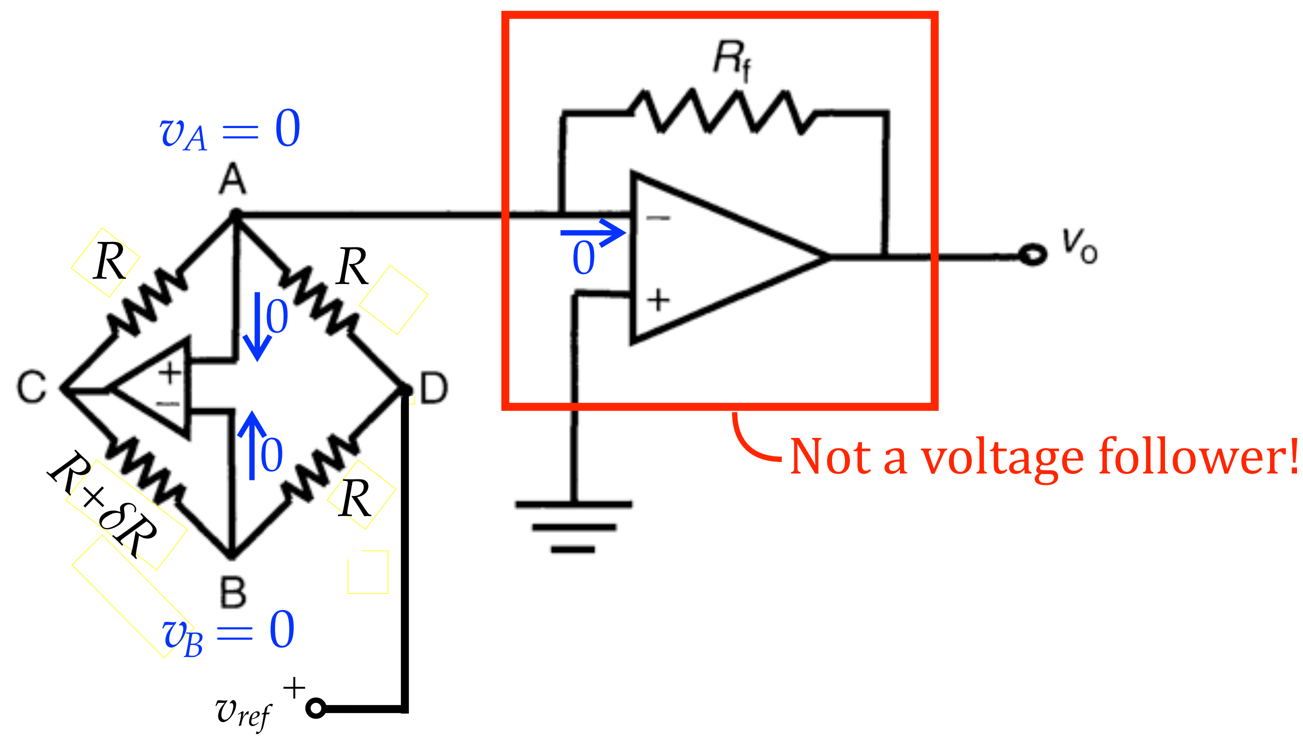
Another Constant-Voltage Resistance Bridge



Or, equivalently:



Another Constant-Voltage Resistance Bridge



Node A: $\dfrac{v_C}{R} + \dfrac{v_{ref}}{R} + \dfrac{v_0}{R_f} = 0$

Node B: $\dfrac{v_{ref}}{R} + \dfrac{v_C}{R+\delta R} = 0 \quad \Rightarrow \quad v_C = -v_{ref}\dfrac{R+\delta R}{R}$

$$\Rightarrow \quad -v_{ref}\frac{R+\delta R}{R^2} + \frac{v_{ref}}{R} + \frac{v_0}{R_f} = 0$$

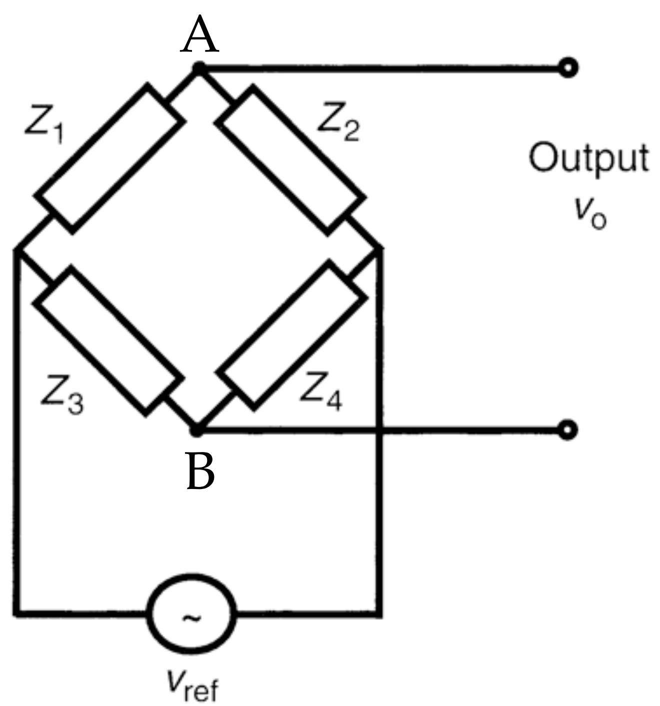
$$\Rightarrow \frac{v_0}{R_f} = v_{ref}\frac{R+\delta R}{R^2} - \frac{v_{ref}}{R} \quad \Rightarrow \quad v_0 = v_{ref}\,R_f\left(\frac{R+\delta R}{R^2} - \frac{1}{R}\right) = v_{ref}\,R_f\left(\frac{R+\delta R}{R^2} - \frac{R}{R^2}\right)$$

$$= v_{ref}\,R_f\left(\frac{\delta R}{R^2}\right)$$

$$= v_{ref}\,\frac{R_f}{R^2}\,\delta R$$

v_o proportional to δR!!

Impedance Bridge



$$V_o(s) = V_A(s) - V_B(s) = \left[\frac{Z_1}{Z_1+Z_2} - \frac{Z_3}{Z_3+Z_4}\right]V_{ref}(s) = \left[\frac{Z_1Z_4 - Z_2Z_3}{(Z_1+Z_2)(Z_3+Z_4)}\right]V_{ref}(s)$$

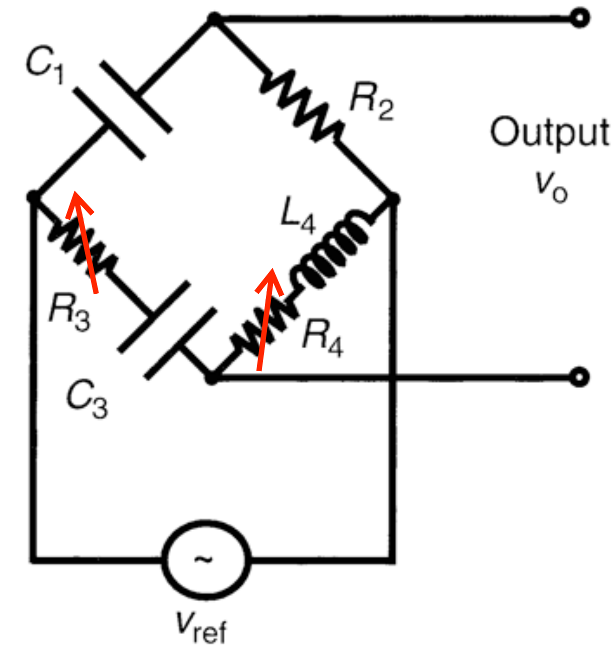
$$= 0 \text{ when } \frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}$$ ← Balanced Bridge Condition

An Owen Bridge is a practical example of an Impedance Bridge

Owen Bridge

$Z_1(s) = \frac{1}{C_1 s}$

$Z_3(s) = R_3 + \frac{1}{C_3 s}$



$Z_2(s) = R_2$

$Z_4(s) = R_4 + L_4 s$

With $v_{ref}(t) = \sin(\omega t)$, balance the bridge (i.e., make $v_o = 0$) by varying R_3 and R_4, then:

$$\frac{Z_1(j\omega)}{Z_2(j\omega)} = \frac{Z_3(j\omega)}{Z_4(j\omega)} \quad \Rightarrow \quad Z_1(j\omega)Z_4(j\omega) = Z_2(j\omega)Z_3(j\omega)$$

$$\Rightarrow \frac{1}{C_1 j\omega}\left[R_4 + L_4 j\omega\right] = R_2\left[R_3 + \frac{1}{C_3 j\omega}\right]$$

$$\Rightarrow \frac{L_4}{C_1} = R_2 R_3 \qquad \text{and} \qquad \frac{R_4}{C_1} = \frac{R_2}{C_3}$$

$$\Rightarrow L_4 = C_1 R_2 R_3 \qquad \text{and} \qquad C_3 = \frac{R_2 C_1}{R_4}$$

$\Rightarrow$ L_4 and C_3 can be determined if C_1, R_2, R_3 and R_4 are known