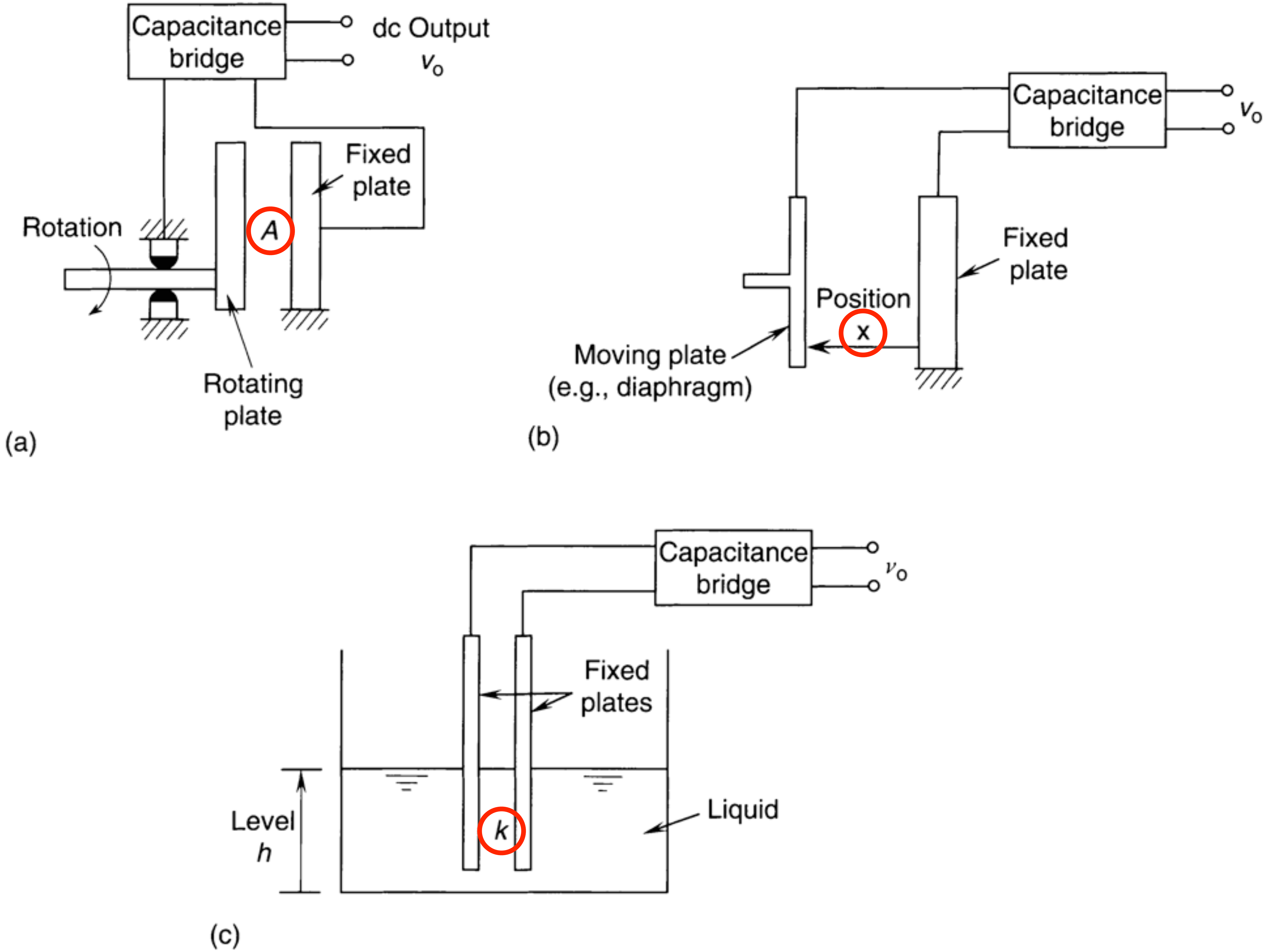
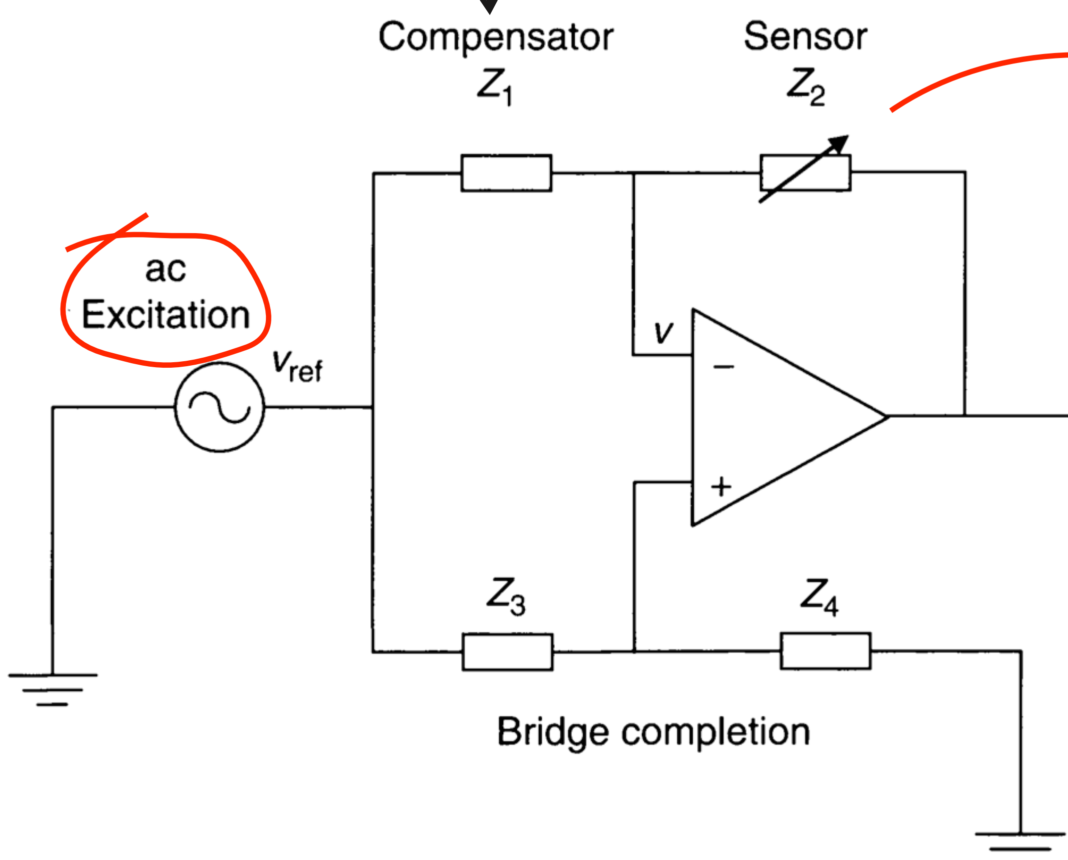


Figure 5.23

Schematic diagrams of capacitive sensors. (a) Capacitive rotation sensor. (b) Capacitive displacement sensor. (c) Capacitive liquid level sensor.

Capacitance Bridge (Sec. 5.6.1.1)

varies similarly to sensor
due to changes in
temperature, humidity, etc.



$$Z_i = \frac{1}{C_i s}$$

$\boxed{Z_2 \text{ is } \textit{inversely} \text{ proportional to } C_2}$

Figure 5.24
A bridge circuit for capacitive sensors.

zero current into op-amp inputs gives:

$$(v_{ref} - v)/Z_1 + (v_o - v)Z_2 = 0$$

$$(v_{ref} - v)/Z_3 + (0 - v)Z_4 = 0$$

$$\Longrightarrow \quad v_o = \frac{(Z_4/Z_3 - Z_2/Z_1)}{1 + Z_4/Z_3} v_{ref}$$

under balanced bridge condition,

$$\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4} \;\Rightarrow\; v_o = 0$$

$$Z_2 \to Z_2 + \delta Z \quad \Rightarrow \boxed{\delta v_o = -\frac{v_{ref}}{Z_1(1 + Z_4/Z_3)}\delta Z}$$

With the amplitude and frequency of v_{ref} constant, the change in the amplitude of v_o will be *inversely* proportional to the change in the capacitance C_2.

Circuit to linearize response of a capacitive displacement sensor

$$i_{Cref}(t) = C_{ref}\frac{dv_{Cref}}{dt} = C_{ref}\frac{dv_{ref}}{dt} \qquad (1)$$

$$i_C(t) = \frac{d(Cv_C)}{dt} = \frac{d(Cv_o)}{dt} \qquad (2)$$

At node A:

$$i_{Cref}(t) + i_C(t) = 0$$

$$\Rightarrow \int_0^t \left[i_{Cref}(\lambda) + i_C(\lambda)\right] d\lambda = 0$$

$$\Rightarrow \int_0^t i_{Cref}(\lambda)\, d\lambda + \int_0^t i_C(\lambda)\, d\lambda = 0 \qquad (3)$$

From (3), using (1) and (2),

$$\int_0^t C_{ref}\, dv_{ref} + \int_0^t d(Cv_o) = 0 \qquad (4)$$

$$\Rightarrow C_{ref}\left[v_{ref}(t) - v_{ref}(0)\right] + \left[C(t)\, v_o(t) - C(0)\, v_o(0)\right] = 0$$

From (4), provided $v_{ref}(0) = v_o(0) = 0$,

$$C_{ref}\, v_{ref}(t) + C(t)\, v_o(t) = 0$$ ← the "charge balance equation for node A" in Sec 5.6.2

$$\Rightarrow v_o(t) = -\frac{C_{ref}\, v_{ref}(t)}{C(t)} \Rightarrow \boxed{v_o(t) = -\frac{C_{ref}\, v_{ref}(t)}{K}\, x(t)}$$ ← $v_o(t)$ is proportional to $x(t)$!!

problem: change in capacitance due to displacement Δx is nonlinear in Δx:

$$\Delta C = kA\left[\frac{1}{x+\Delta x} - \frac{1}{x}\right]$$

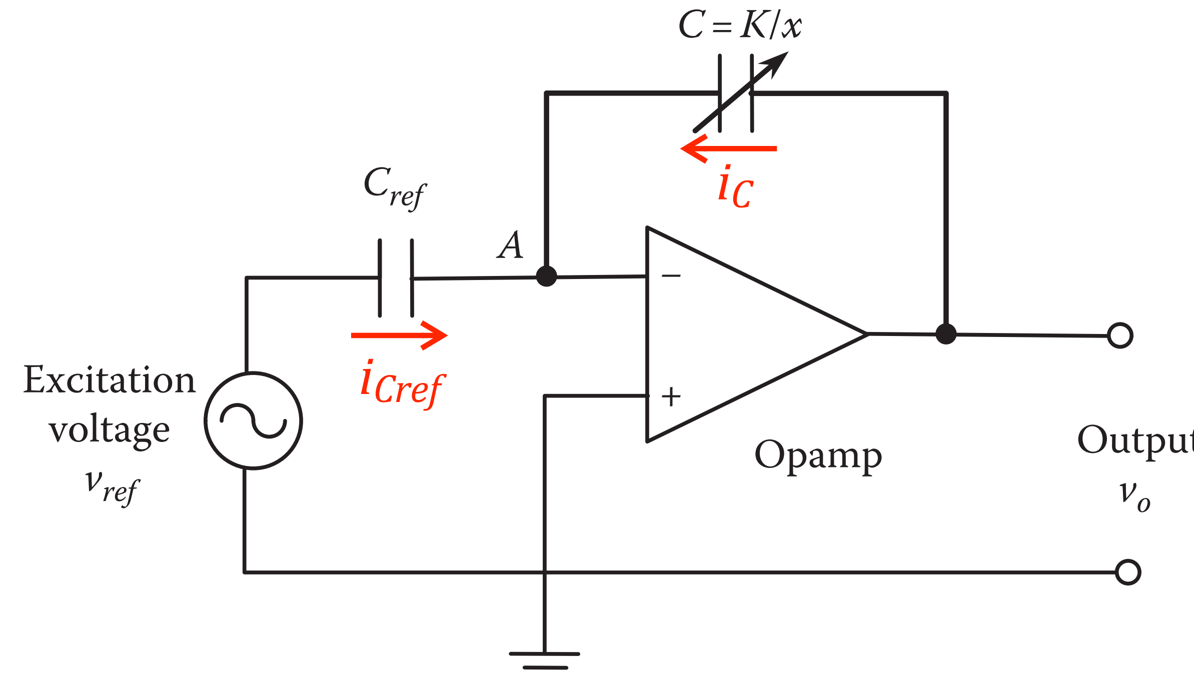
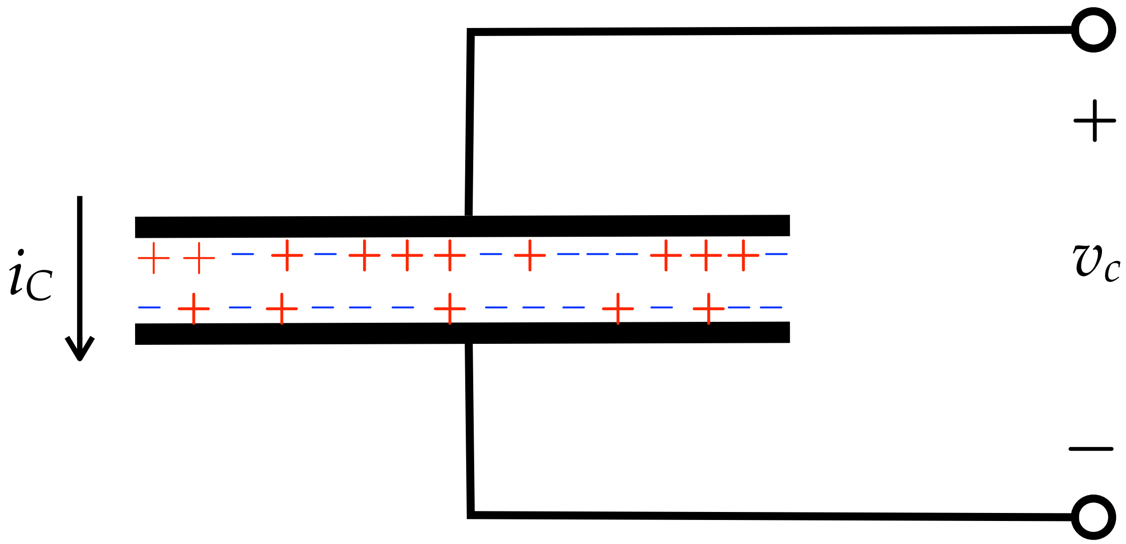


Figure 5.27
Linearizing amplifier circuit for a capacitive transverse displacement sensor.

Piezoelectric Sensors

Capacitor fundamentals:



Current $=$ $C * $ Rate of change of voltage

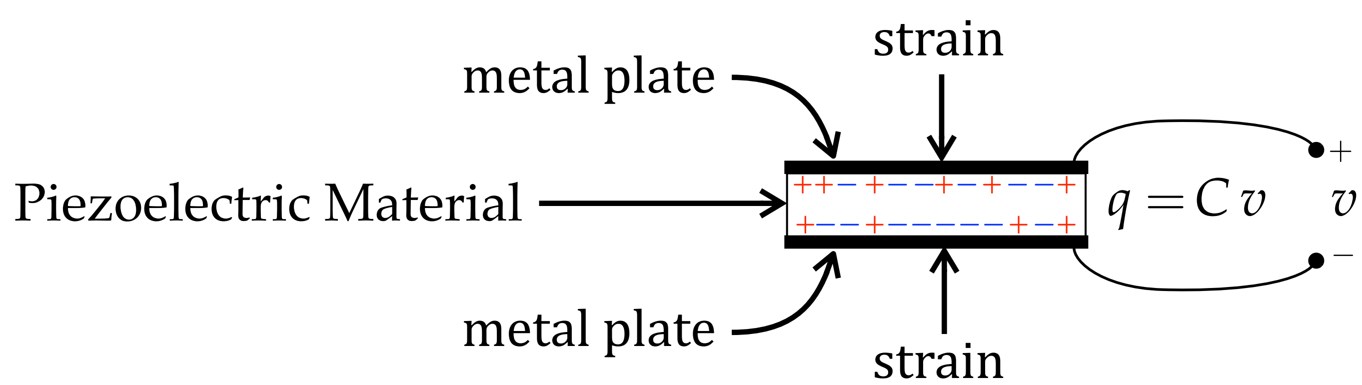
$$\boxed{i_c = C\frac{dv_c}{dt}} \Leftrightarrow \quad v_c(t) = \frac{1}{C}\int_{-\infty}^{t} i_c(\lambda)\,d\lambda$$

$$\boxed{i_c = \frac{dq_c}{dt}} \quad \Leftrightarrow \quad q_c(t) = \int_{-\infty}^{t} i_c(\lambda)\,d\lambda$$

Current $=$ Rate of change of charge

$$\begin{aligned} \text{Net charge on capacitor} &= \text{Net positive charge on positive plate} - \text{Net positive charge on negative plate} \\ &= C\,v_c \\ &= q_c \end{aligned}$$

The Piezoelectric Effect



Some substances generate an electrical charge and an associated potential difference when they are subjected to mechanical stress or strain. This *piezoelectric effect* is used in piezoelectric sensors.

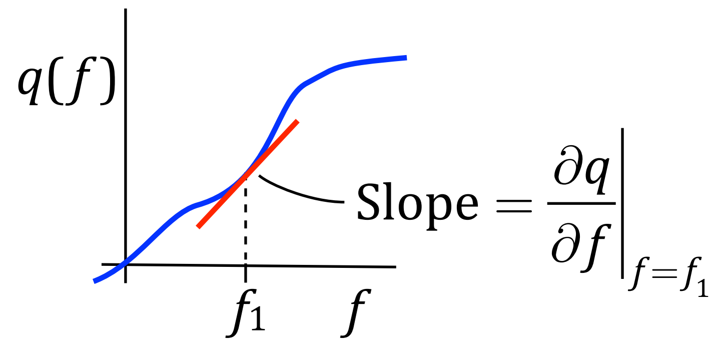
Conversely, piezoelectric materials deform when subject to a potential difference. This is the reverse piezoelectric effect. It is used to create piezoelectric actuators.

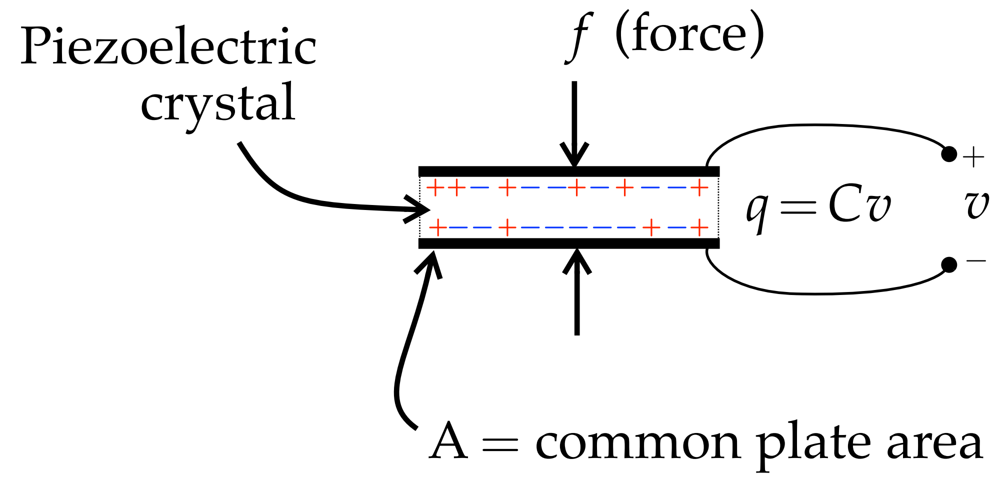
Sensitivity of Piezoelectric Crystals

Charge Sensitivity:

$$S_q \triangleq \frac{\partial q}{\partial f} = \frac{\Delta\text{charge}}{\Delta\text{force}}$$

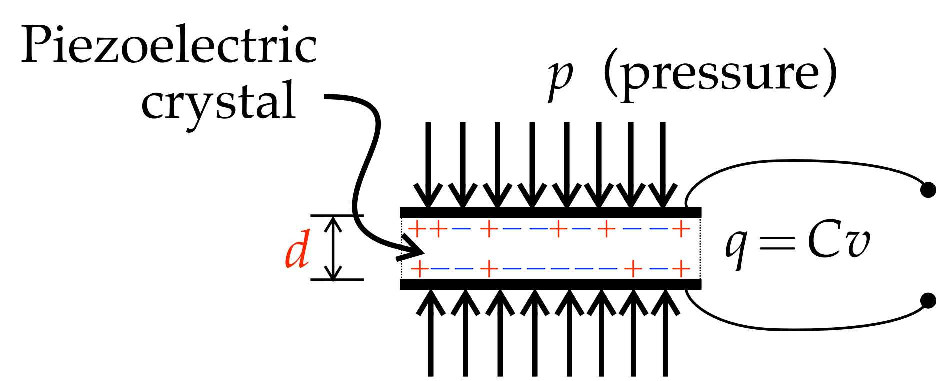
$$= \frac{\partial q}{A\,\partial p} = \frac{\Delta\text{charge}}{A \times \Delta\text{pressure}}$$





Voltage Sensitivity:

$$S_v \triangleq \frac{\partial v}{d\,\partial p} = \frac{\Delta\text{voltage}}{\Delta\text{pressure}}$$ per unit crystal thickness



Assuming that C is a constant

$$q = Cv \;\Rightarrow\; \delta q = C\,\delta v$$

and it can be shown that

$$S_q = k S_v$$ } Derivation is a forthcoming homework problem

where k is the dielectric constant for the crystal.

Table 5.5 Direct Sensitivities of Several Piezoelectric Materials

Material	Charge Sensitivity S_q (pC/N)	Voltage Sensitivity S_v (mV.m/N)
Lead Zirconate Titanate (PZT)	110	10
Barium Titanate	140	6
Quartz	2.5	50
Rochelle Salt	275	90