Strain Gages

5.8.1 Equations for Strain-Gage Measurements

The change of electrical resistance in material when mechanically deformed is the property used in resistance-type strain gages. The resistance R of a conductor that has length ℓ and area of cross-section A is given by

$$R = \rho \frac{\ell}{A}, \tag{5.51}$$

where ρ denotes the resistivity of the material. Taking the logarithm of Equation 5.51, we have

$$\log R = \log \rho + \log(\ell/A).$$

Fractional change in resistance
Fractional change in resistivity
Fractional change in length/area

Now, taking the differential of each term, we obtain

$$\frac{\mathrm{d}R}{R} = \frac{\mathrm{d}\rho}{\rho} + \frac{\mathrm{d}(\ell/A)}{\ell/A}. \tag{5.52}$$

The first term on the RHS of Equation 5.52 is the fractional change in resistivity, and the fractional second term represents deformation. It follows that the change in resistance in the material comes from the change in shape as well as from the change in resistivity of the material. For linear deformations, the two terms on the RHS of Equation 5.52 are linear functions of strain ε; the proportionality constant of the second term, in particular, depends on Poisson's ratio of the material. Hence, the following relationship can be written for a strain-gage element:

$$\boxed{\frac{\delta R}{R} = S_s \varepsilon} \tag{5.53}$$

The constant S_s is known as the gage factor or sensitivity of the strain-gage element. The numerical value of this parameter ranges from 2 to 6 for most metallic strain-gage elements and from 40 to 200 for semiconductor strain gages.

$$\frac{\delta R}{R} = S_s \varepsilon$$

S_s

Table 5.6

Properties of Common Strain-Gage Material

	Material	Composition	Gage Factor (Sensitivity)	Temperature Coefficient of Resistance ($10^{-6}/°C$)
Metal Foil	Constantan	45% Ni, 55% Cu	2.0	15
	Isoelastic	36% Ni, 52% Fe, 8% Cr, 4% (Mn, Si, Mo)	3.5	200
	Karma	74% Ni, 20% Cr, 3% Fe, 3% Al	2.3	20
	Monel	67% Ni, 33% Cu	1.9	2000
Semiconductor	Silicon	p-type	100 to 170	70 to 700
	Silicon	n-type	−140 to −100	70 to 700

Undesirable characteristics of SC gages include the following:

1. The strain–resistance relationship is more nonlinear.
2. They are brittle and difficult to mount on curved surfaces.
3. The maximum strain that can be measured is one to two orders of magnitude smaller (typically, less than 0.001 m/m).
4. They are more costly.
5. They have much larger temperature sensitivity.

From

$$\frac{\delta R}{R} = S_s \varepsilon \;\Rightarrow\; \delta R = R\, S_s \varepsilon$$

for fixed gage size, a "serpentine" resistive element increases R, which increases δR, for the same ε.

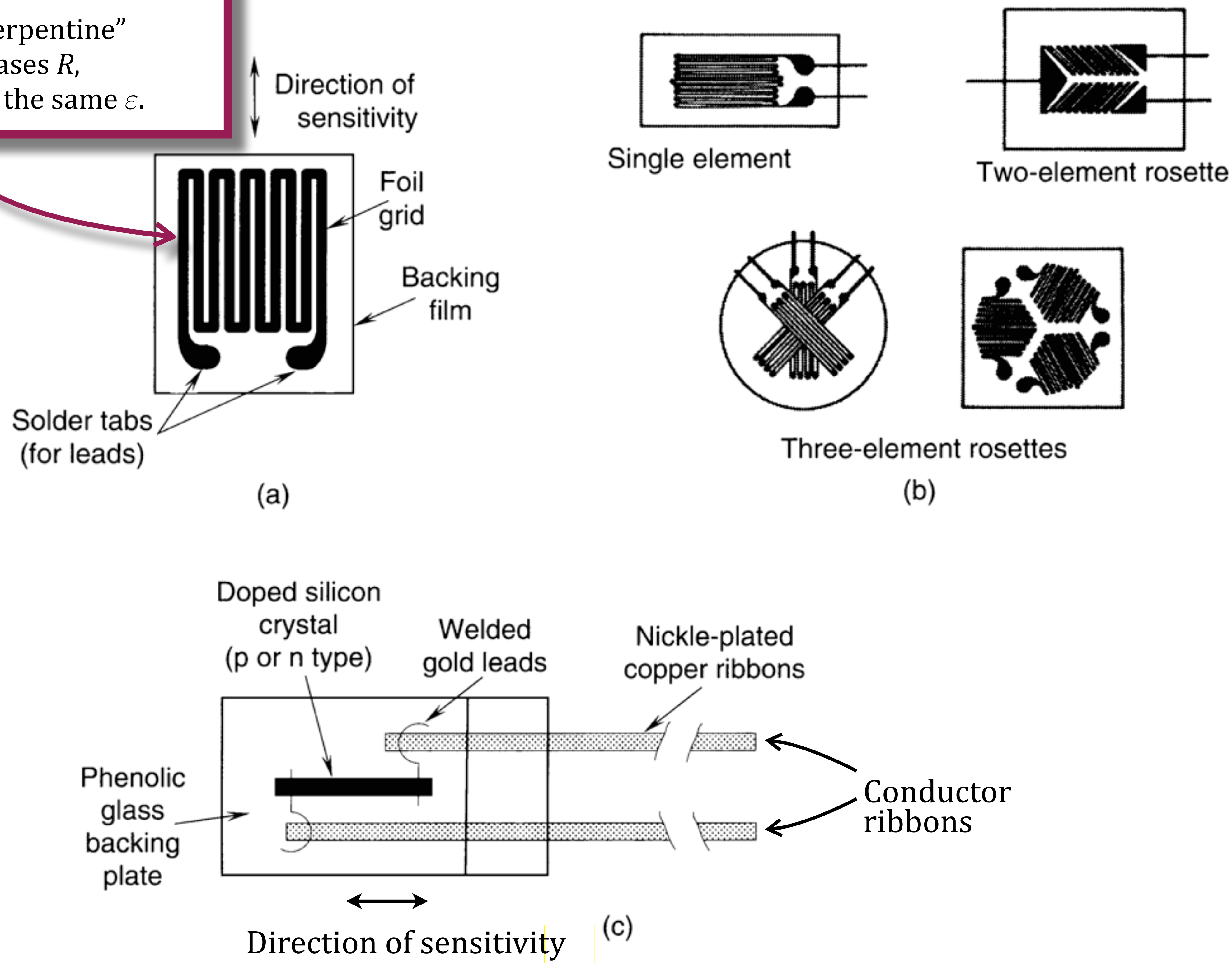


Figure 5.36

(a) Strain-gage nomenclature. (b) Typical foil-type strain gages. (c) A semiconductor strain-gage.

Many applications, for example:

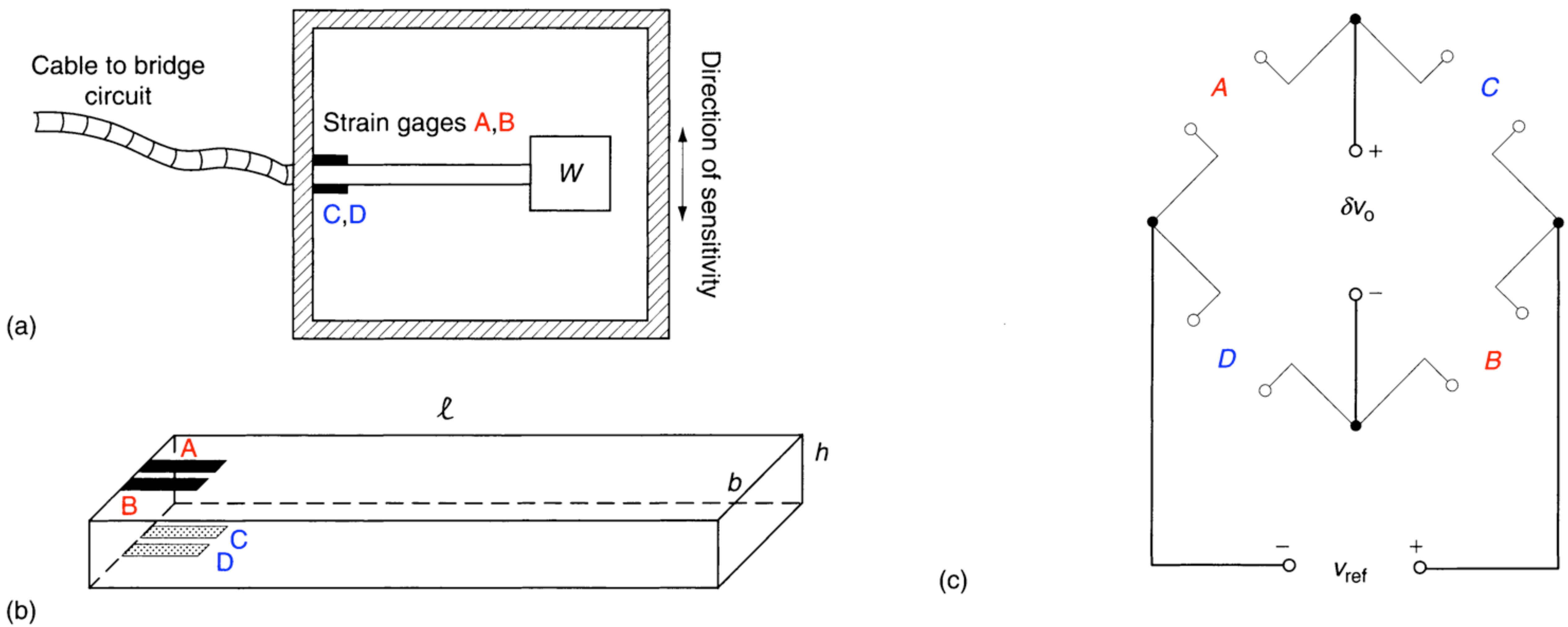
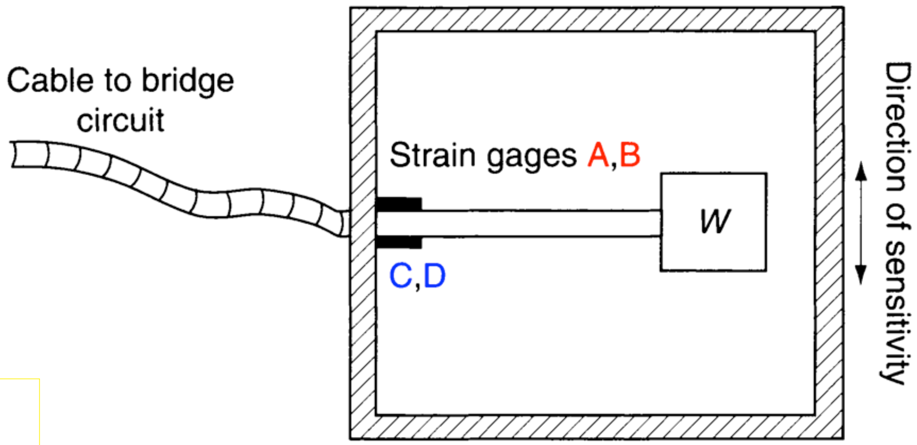
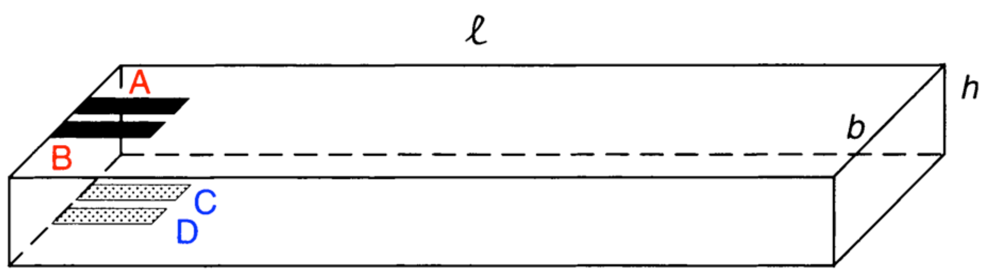
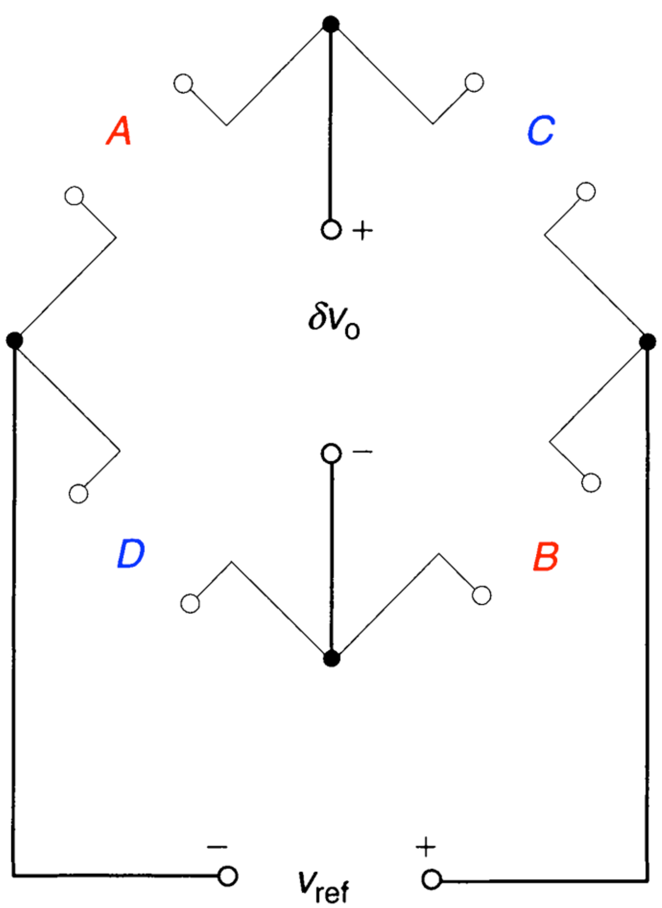


Figure 5.35
A miniature accelerometer using strain gages: (a) Schematic diagram; (b) Mounting configuration of the strain gages; (c) Bridge connection.







The four gages are identical and symmetrically located, so

$$R_A = R + \delta R \quad R_B = R + \delta R \quad R_C = R - \delta R \quad R_D = R - \delta R$$

As a consequence of the placement of the four gages in the bridge circuit,

$$\frac{\delta v_o}{v_{ref}} = \frac{\delta R_A + \delta R_B - \delta R_C - \delta R_D}{4R} = \frac{\delta R + \delta R - (-\delta R) - (-\delta R)}{4R} = \frac{\delta R}{R}$$

Each of the strain gages satisfies

$$\frac{\delta R}{R} = S_s\,\varepsilon$$

Gage factor (a.k.a. gage sensitivity)

Strain

So for the complete system

$$\boxed{\frac{\delta v_o}{v_{ref}} = S_s\,\varepsilon}$$

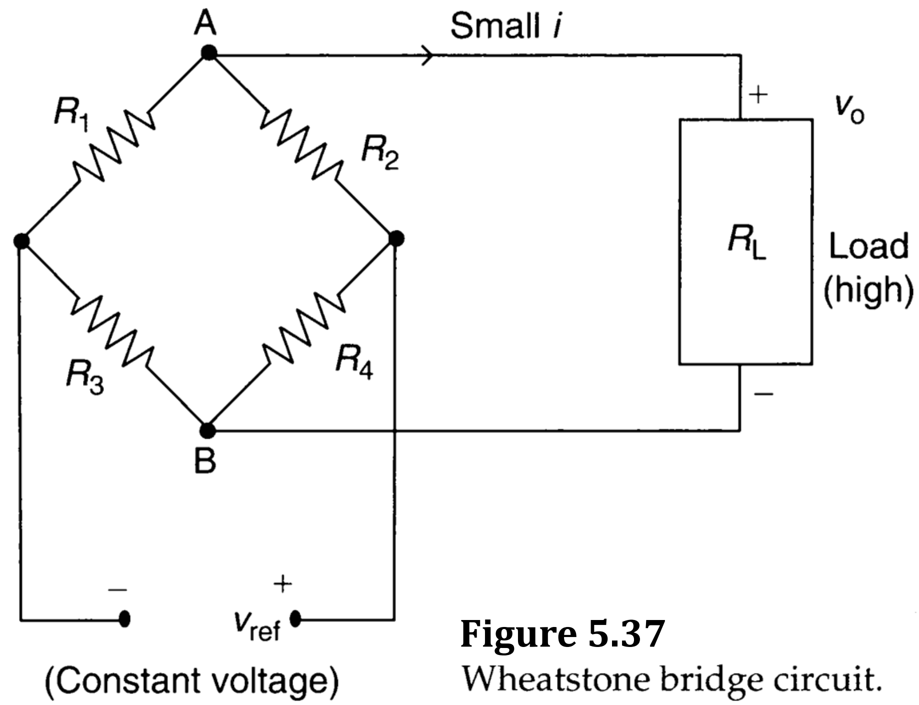


Figure 5.37
Wheatstone bridge circuit.

5.8.1.1 Bridge Sensitivity

Strain-gauge measurements are calibrated with respect to a balanced bridge. When a strain gauge in the bridge deforms, the balance is upset. If one of the arms of the bridge has a variable resistor, it can be adjusted to restore the balance. The amount of this adjustment measures the amount by which the resistance of the strain gauge has changed, thereby measuring the applied strain. This is known as the *null-balance method* of strain measurement. This method is inherently slow because of the time required to balance the bridge each time a reading is taken. A more common method, which is particularly suitable for making dynamic readings from a strain-gauge bridge, is to measure the output voltage resulting from the imbalance caused by the deformation of an active strain gauge in the bridge. To determine the calibration constant of a strain-gauge bridge, the sensitivity of the bridge output to changes in the four resistors in the bridge should be known. For small changes in resistance, using straightforward calculus, this may be determined as

$$\frac{\delta v_0}{v_{ref}} = \frac{(R_2\delta R_1 - R_1\delta R_2)}{(R_1 + R_2)^2} - \frac{(R_4\delta R_3 - R_3\delta R_4)}{(R_3 + R_4)^2} \tag{5.56}$$

This result is subject to Equation 5.55, because changes are measured from the balanced condition. Note from Equation 5.56 that if all four resistors are identical (in value and material), the changes in resistance due to ambient effects cancel out among the first-order terms (δR_1, δR_2, δR_3, δR_4), producing no net effect on the output voltage from the bridge. Closer examination of Equation 5.56 reveals that only the adjacent pairs of resistors (e.g., R_1 with R_2, and R_3 with R_4) have to be identical in order to achieve this environmental compensation. Even this requirement can be relaxed. In fact, compensation is achieved if R_1 and R_2 have the same temperature coefficient and if R_3 and R_4 have the same temperature coefficient.

e.g., a change in the ambient temperature

Torque Sensors

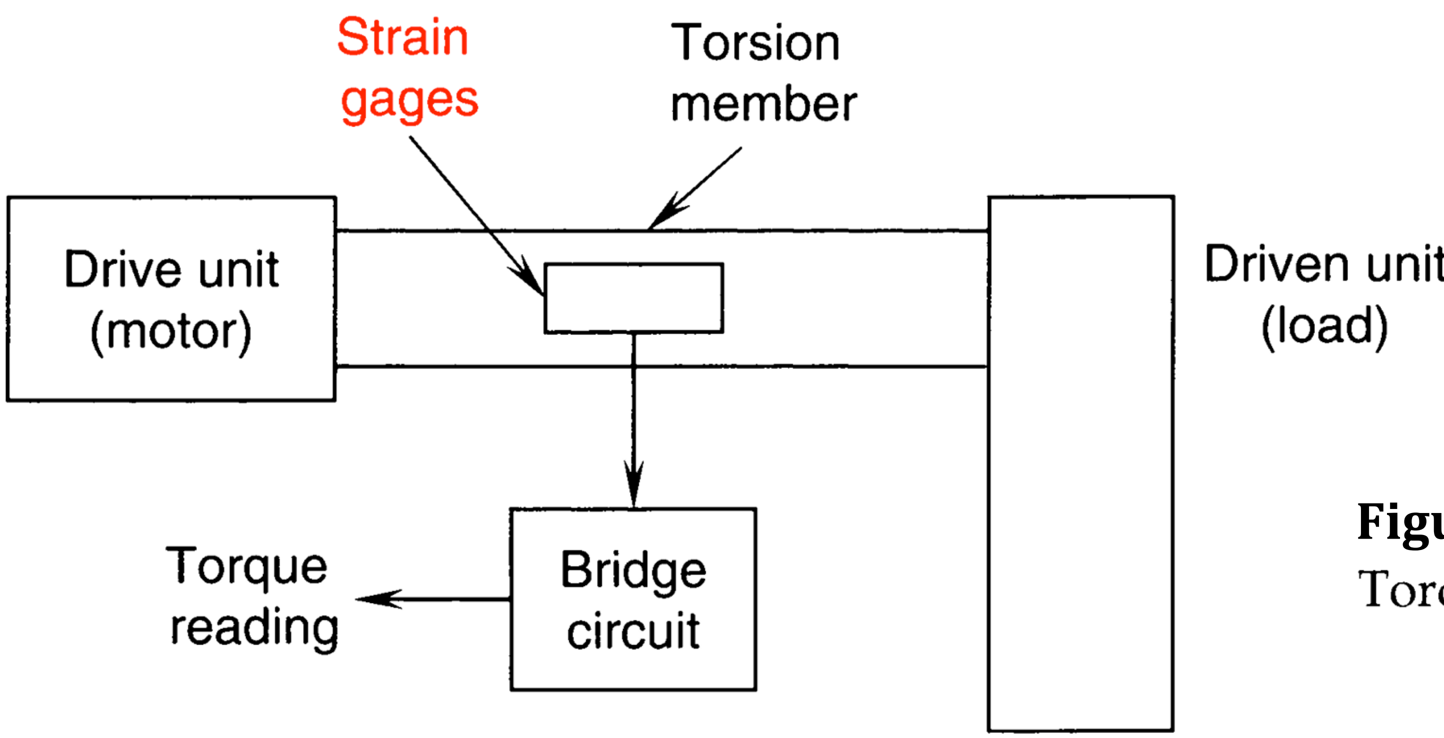


Figure 5.46
Torque sensing using a torsion member.

5.9.1 Strain-Gage Torque Sensors

The most straightforward method of torque sensing is to connect a torsion member between the drive unit and the (driven) load in series, as shown in Figure 5.46, and to measure the torque in the torsion member.

If a circular shaft (solid or hollow) is used as the torsion member, the torque–strain relationship becomes relatively simple, and is given by:

$$\varepsilon = \frac{r}{2GJ}T, \tag{5.66}$$

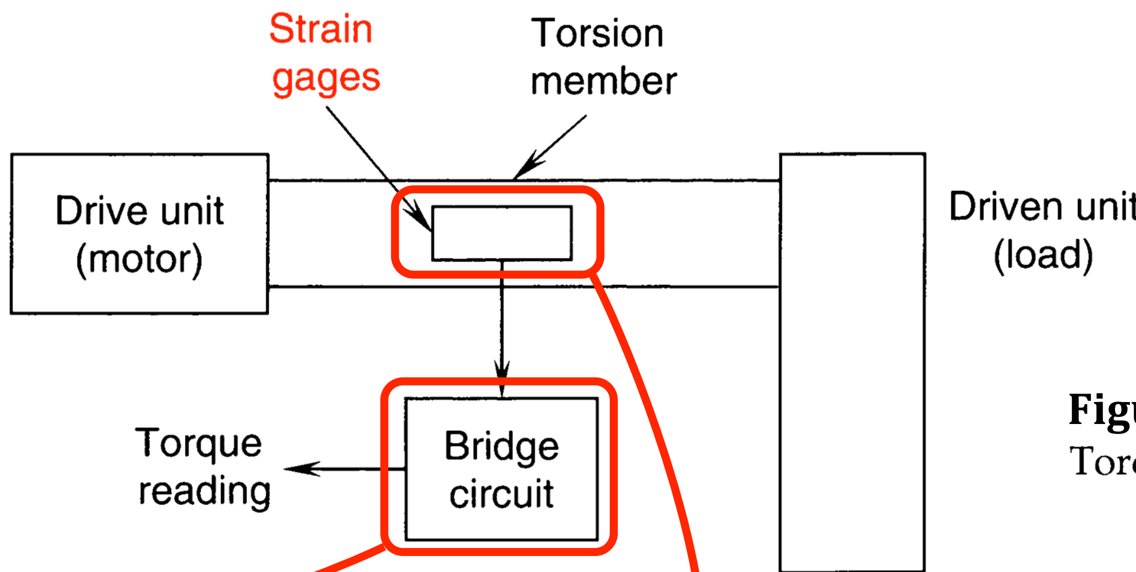
where T is the torque transmitted through the member, ε is the principal strain (which is at 45° to shaft axis) at radius r within the member, J is the polar moment of area of cross-section of the member, and G is the shear modulus of the material.

Torque Sensors

For each strain gage:

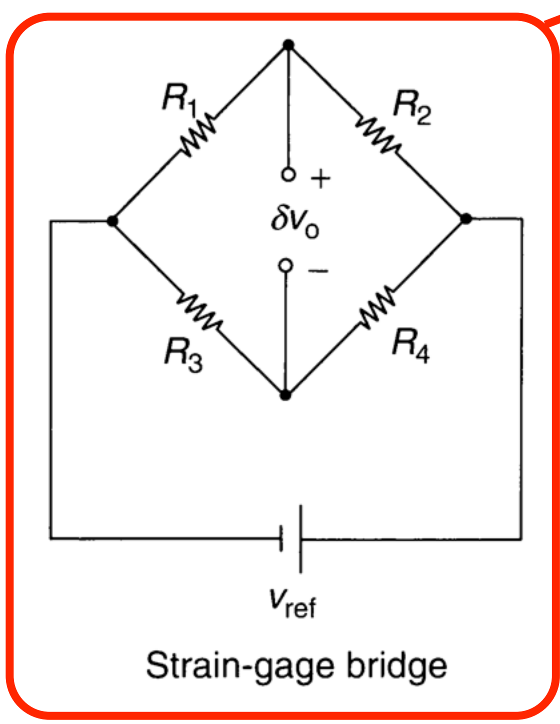
$$\frac{\delta R}{R} = S_S\, \varepsilon \qquad (2)$$

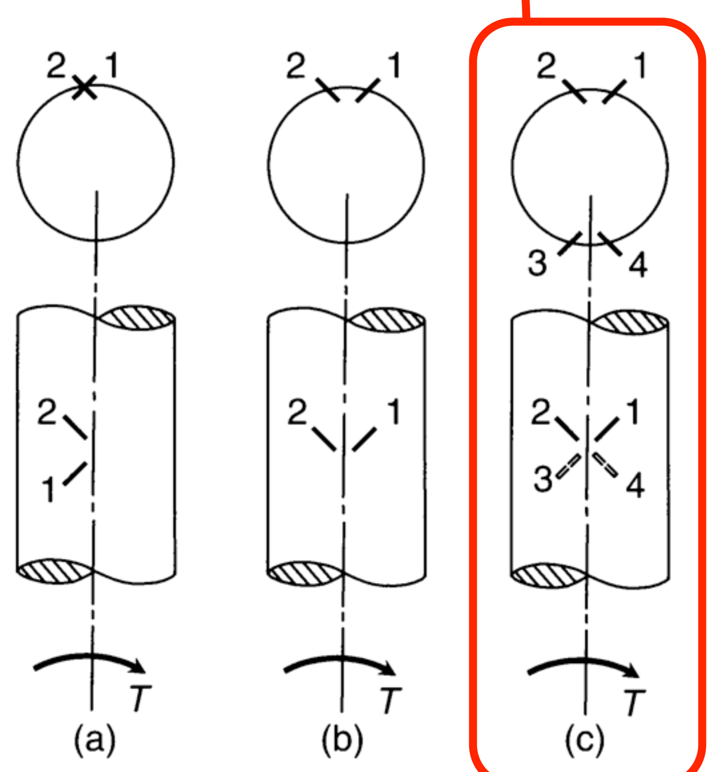
where S_S is the gage factor (sensitivity) of the gage and ε is the strain seen by the gage.



$$\varepsilon = \frac{r}{2GJ}\, T \qquad (1)$$

Figure 5.46
Torque sensing using a torsion member.





With identical gages:

$$R_1 = R + \delta R \quad R_2 = R - \delta R$$
$$R_3 = R - \delta R \quad R_4 = R + \delta R$$

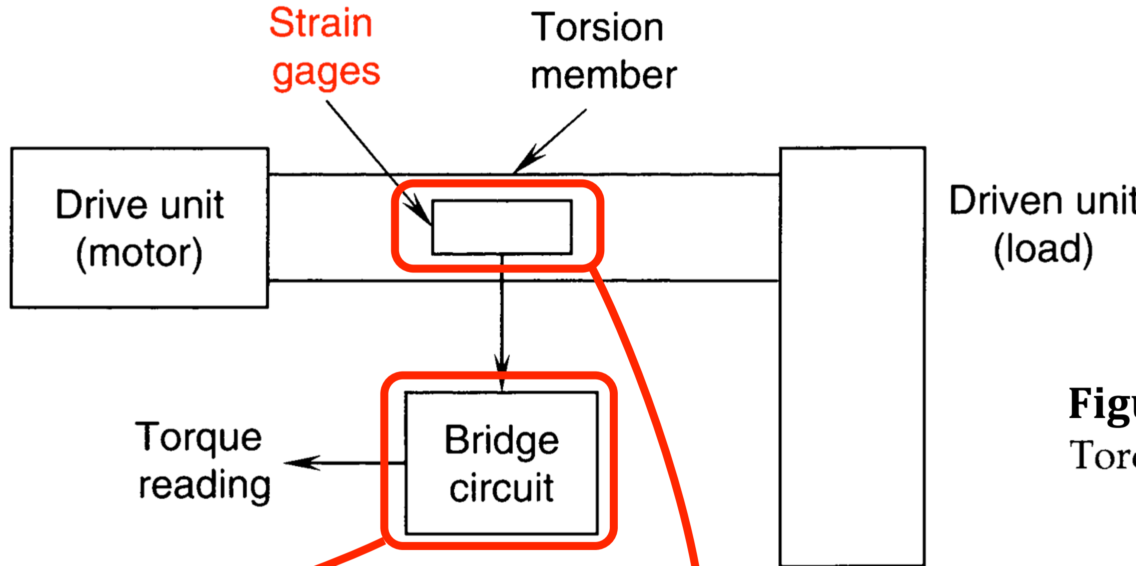
Figure 5.47
Strain-gage configurations for a circular shaft torque sensor.

Torque Sensors

For each strain gage:

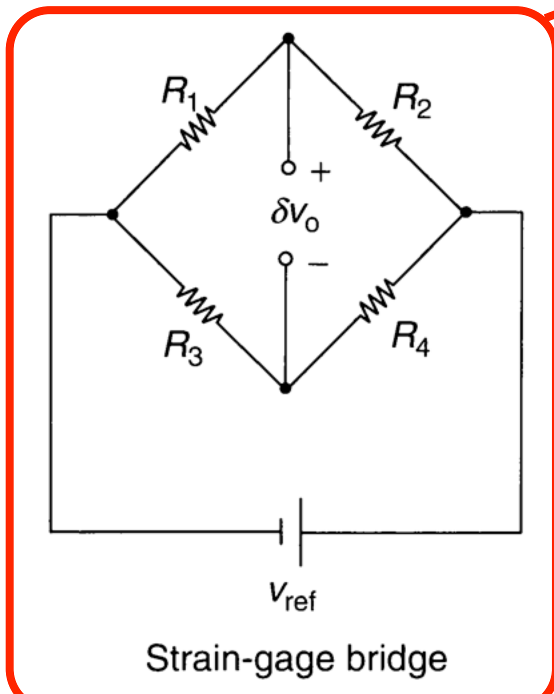
$$\frac{\delta R}{R} = S_S \, \varepsilon \qquad (2)$$

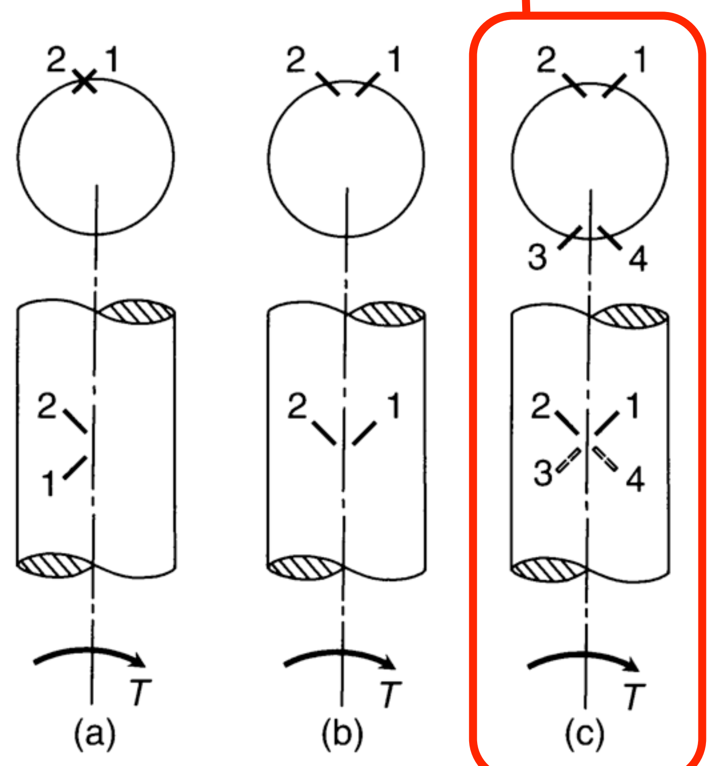
where S_S is the gage factor (sensitivity) of the gage and ε is the strain seen by the gage.



$$\varepsilon = \frac{r}{2GJ} T \qquad (1)$$

Figure 5.46
Torque sensing using a torsion member.





With identical gages:

$$R_1 = R + \delta R \quad R_2 = R - \delta R$$
$$R_3 = R - \delta R \quad R_4 = R + \delta R$$

and

$$\frac{\delta v_o}{v_{ref}} = \frac{\delta R}{R} \qquad (3)$$

From (1), (2) and (3):

$$\boxed{\frac{\delta v_o}{v_{ref}} = S_S \frac{r}{2GJ} T}$$

Configuration	(a)		(b)		(c)
Bridge constant (k)	2		2		4
Axial loads compensated	Yes		Yes		Yes
Bending loads compensated	Yes		Yes		Yes

Factor of sensitivity increase compared to when just one active strain gain is used.

Figure 5.47
Strain-gage configurations for a circular shaft torque sensor.

Loading Effect

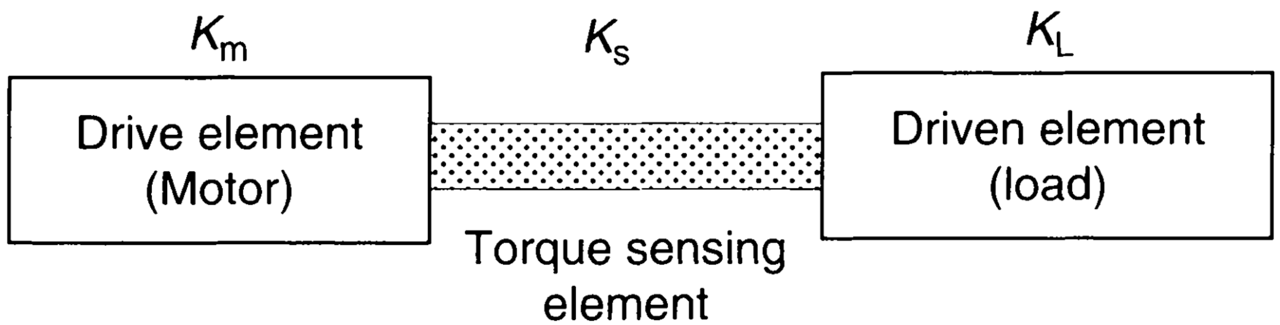


Figure 5.48
Stiffness degradation due to flexibility of the torque-sensing element.

Compliances sum in *series* and the *compliance* of a spring with *stiffness* K is $1/K$, so the *stiffness*, K_{new}, of the Motor, Torque Sensing Element, and Load in series satisfies:

$$\frac{1}{K_{new}} = \frac{1}{K_m} + \frac{1}{K_s} + \frac{1}{K_L}$$

To *minimize* the loading effect we want

$$\frac{1}{K_s} \ll \frac{1}{K_m} + \frac{1}{K_L} \quad \Rightarrow \quad K_s \gg \frac{K_m K_L}{K_m + K_L}$$

But to *maximize* the sensitivity of the Torque Sensing Element we want K_S to be *small*.

Loading Effect

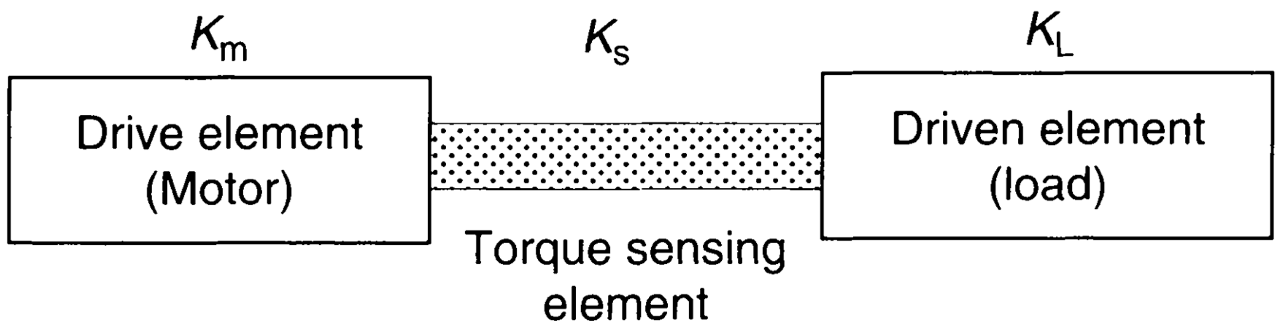


Figure 5.48
Stiffness degradation due to flexibility of the torque-sensing element.

The torsional stiffness of the torque sensing element is shown, in Sec. 5.9.2.4, to be

$$K_S = \frac{GJ}{L}$$

where G is the shear modulus of the material, J is the polar moment of area of the cross-section, and L is the length.

The above expression for K_S and

$$\varepsilon = \frac{r}{2GJ}T$$

can be used to show that, for the same K_S, the sensitivity of the torque sensing element can be increased via use, for the torque sensing element, of a large diameter, thin-walled tube. This, however, gives rise to other issues, e.g., a tendency to buckle under radial loads.