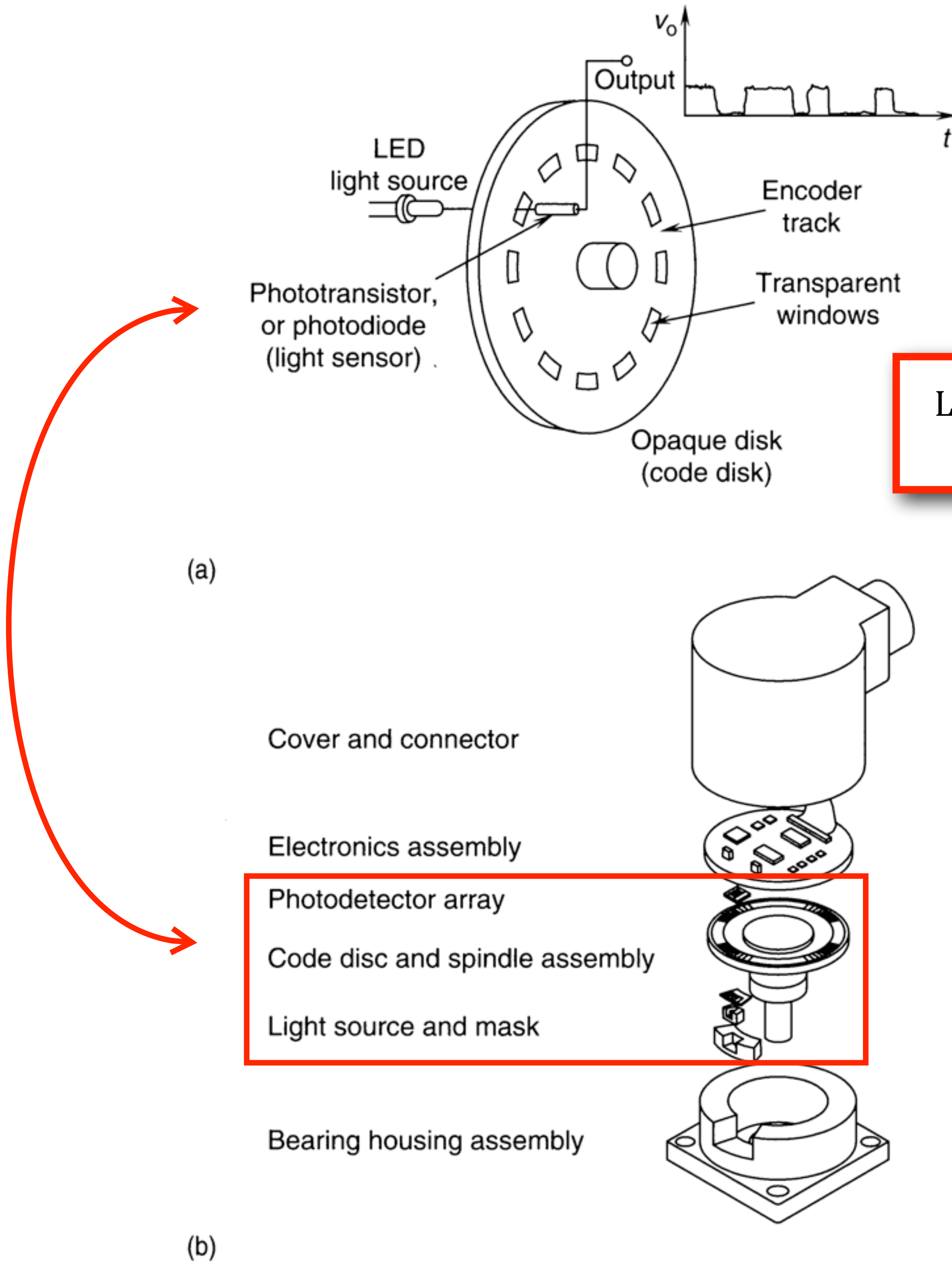
Digital Sensors

Chapter 6

ME 473

Professor Sawyer B. Fuller

Count pulses to deduce angular position of encoder disk and thus of shaft.



LED and phototransistor are *fixed*. Encoder disk *rotates* with shaft.

How to sense the *direction* of rotation?

Figure 6.2
(a) Schematic representation of an (incremental) optical encoder; (b) components of a commercial incremental encoder. (BEI Electronics, Inc. With permission.)

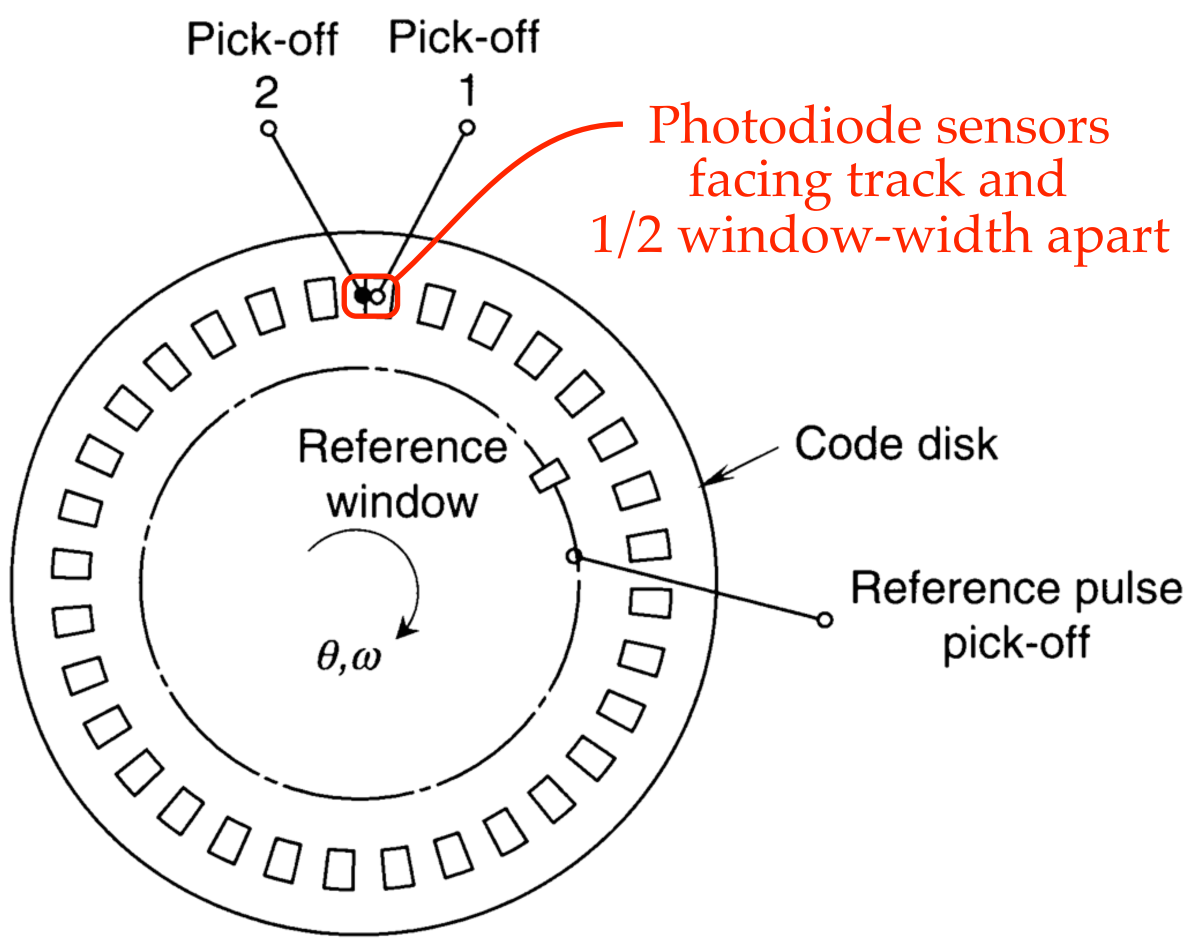


Figure 6.3
An incremental encoder disk (offset sensor configuration).

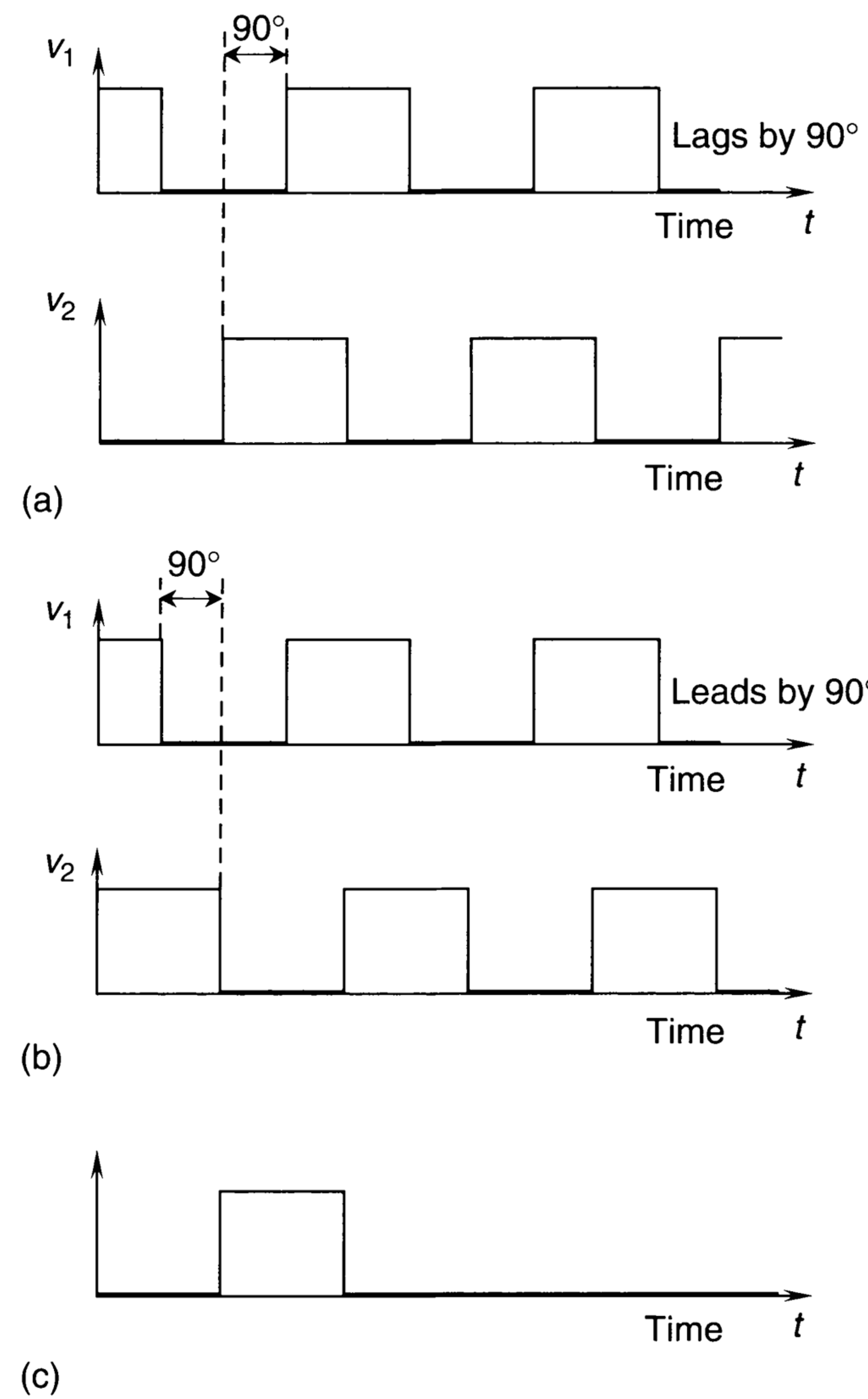
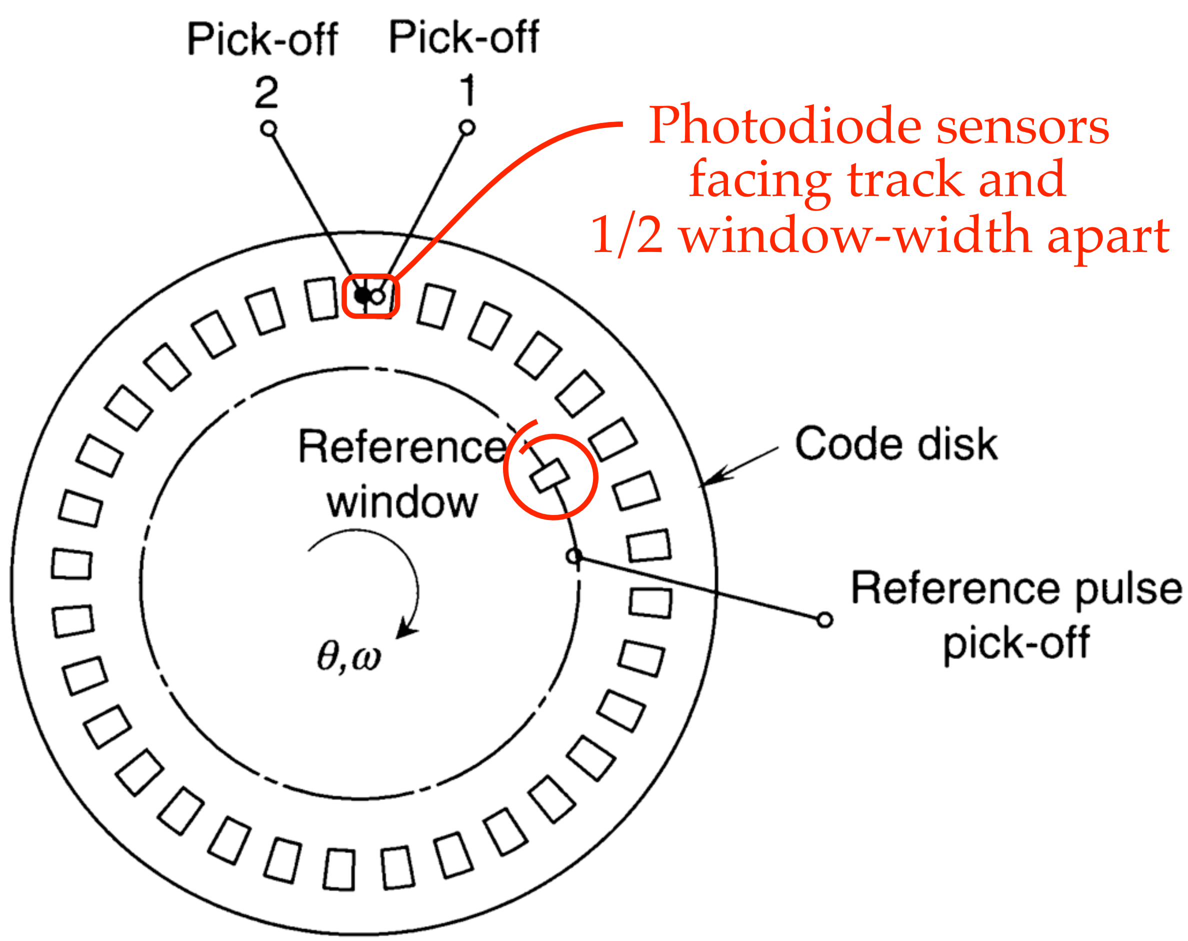


Figure 6.4
Shaped pulse signals from an incremental encoder (a) for clockwise rotation; (b) for counterclockwise rotation; (c) reference pulse signal.



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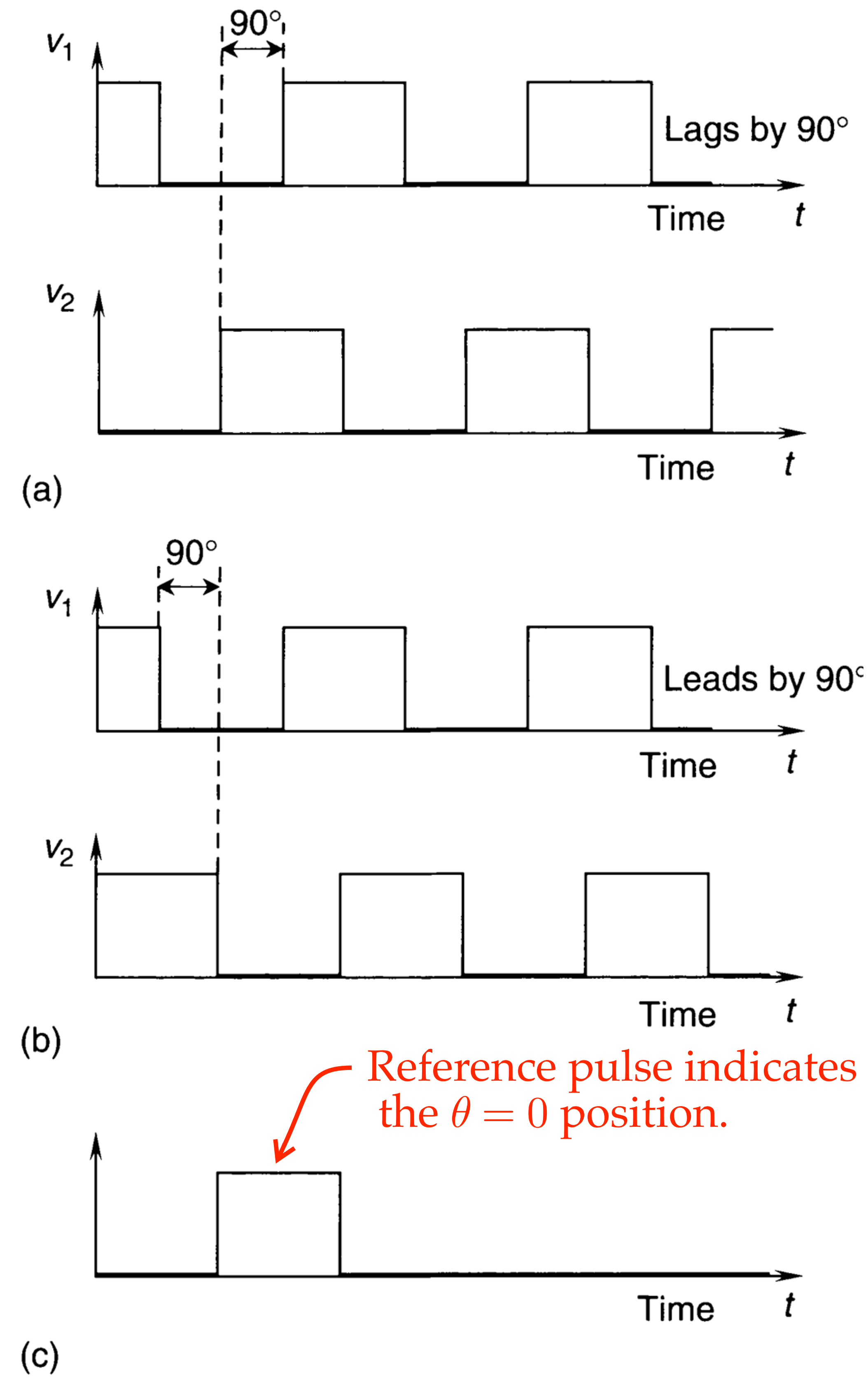

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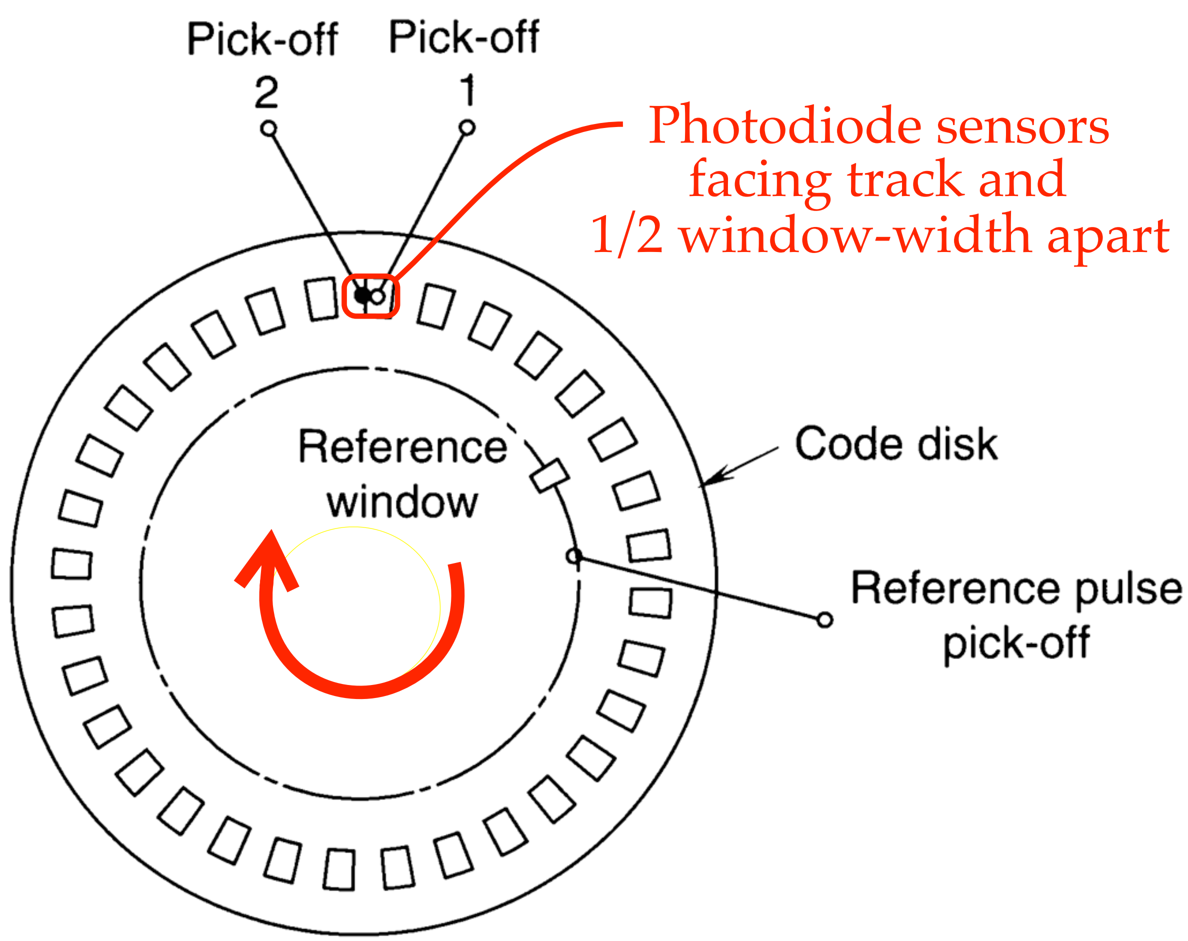

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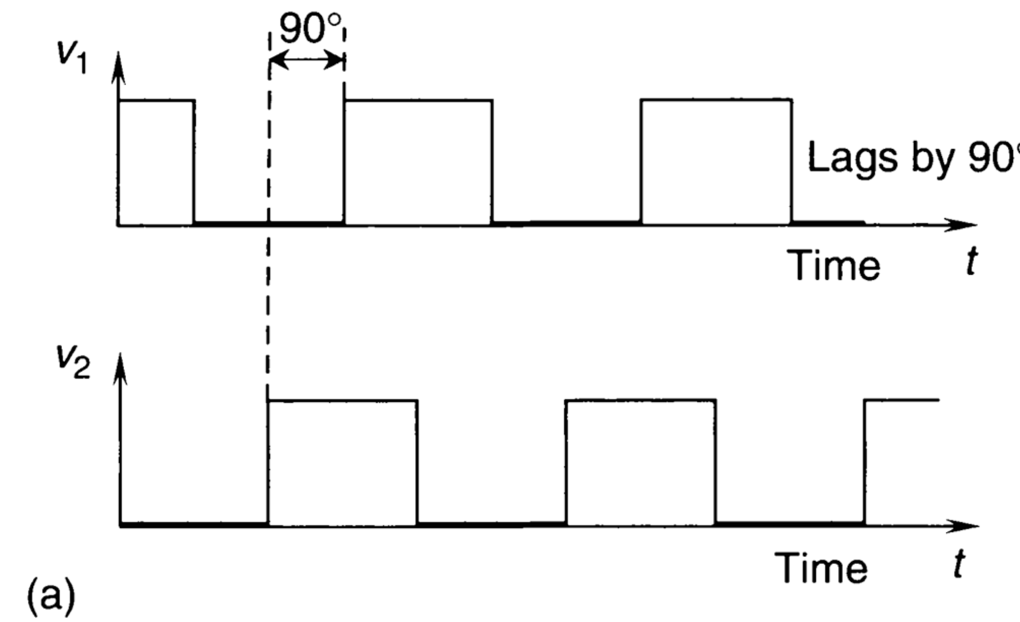
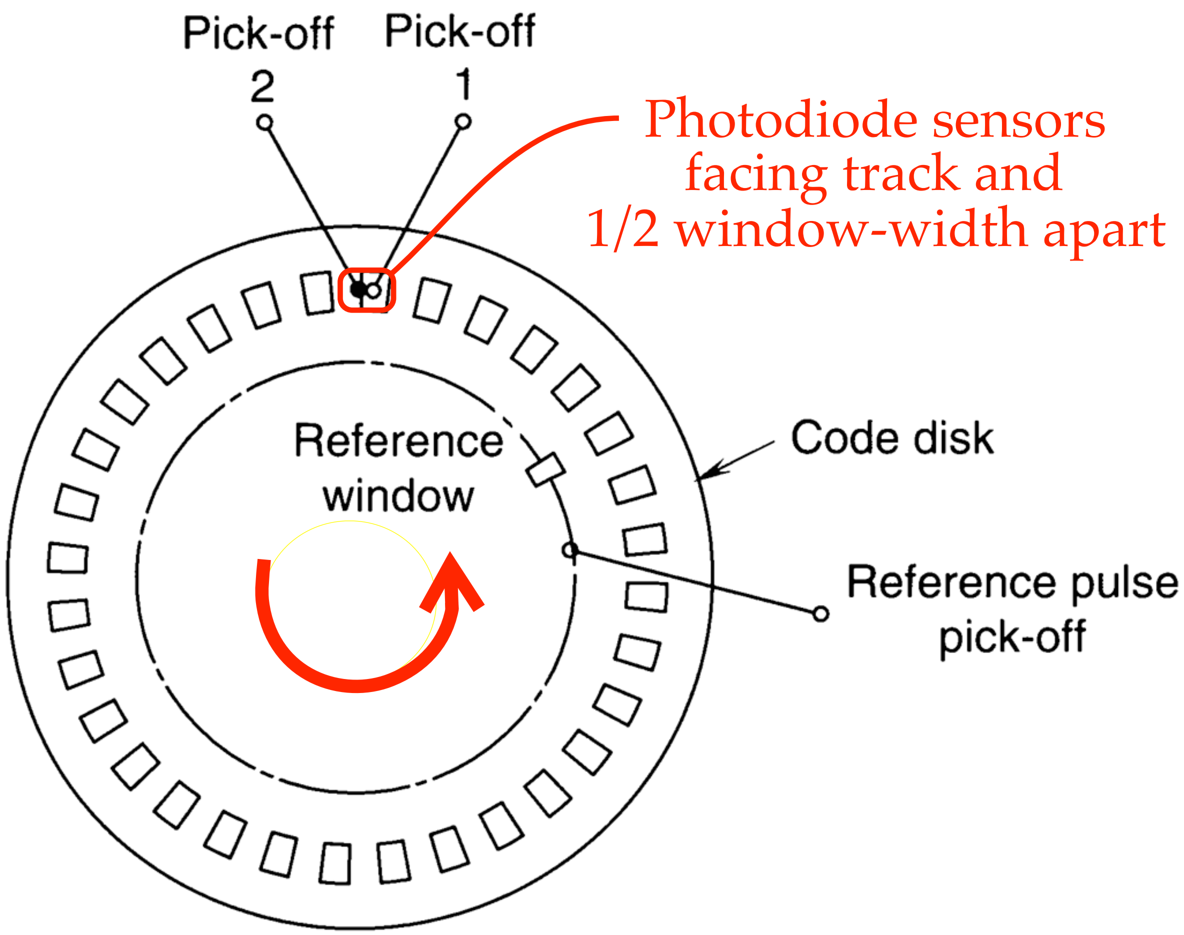


Under clockwise rotation, v_2 goes high first, then, 1/4 cycle (90°) later, v_1 goes high.

Figure 6.4
Shaped pulse signals from an incremental encoder (a) for clockwise rotation; (b) for counterclockwise rotation; (c) reference pulse signal.



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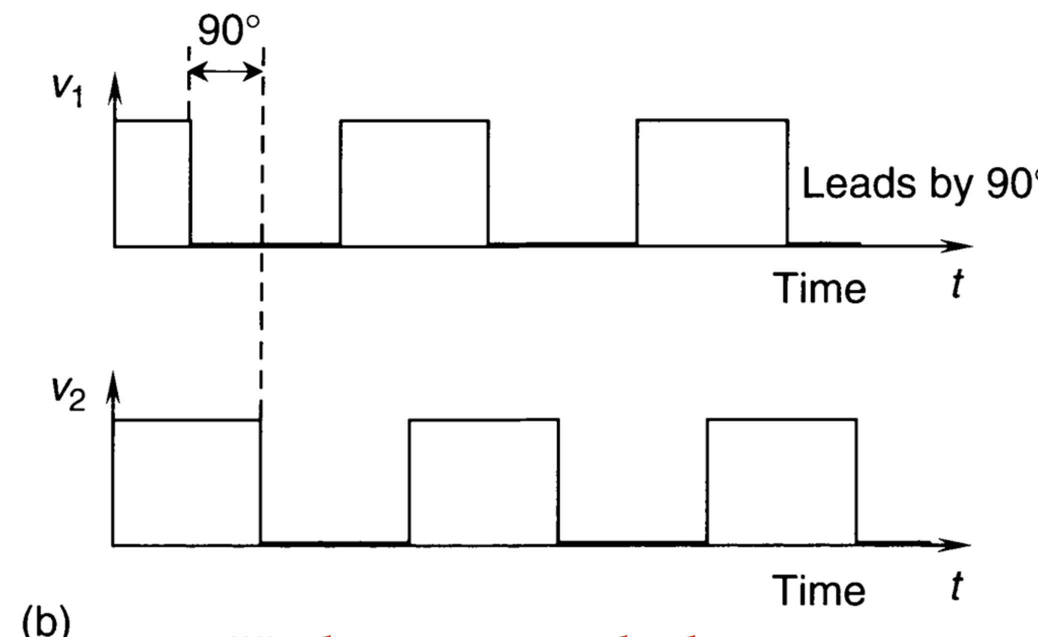
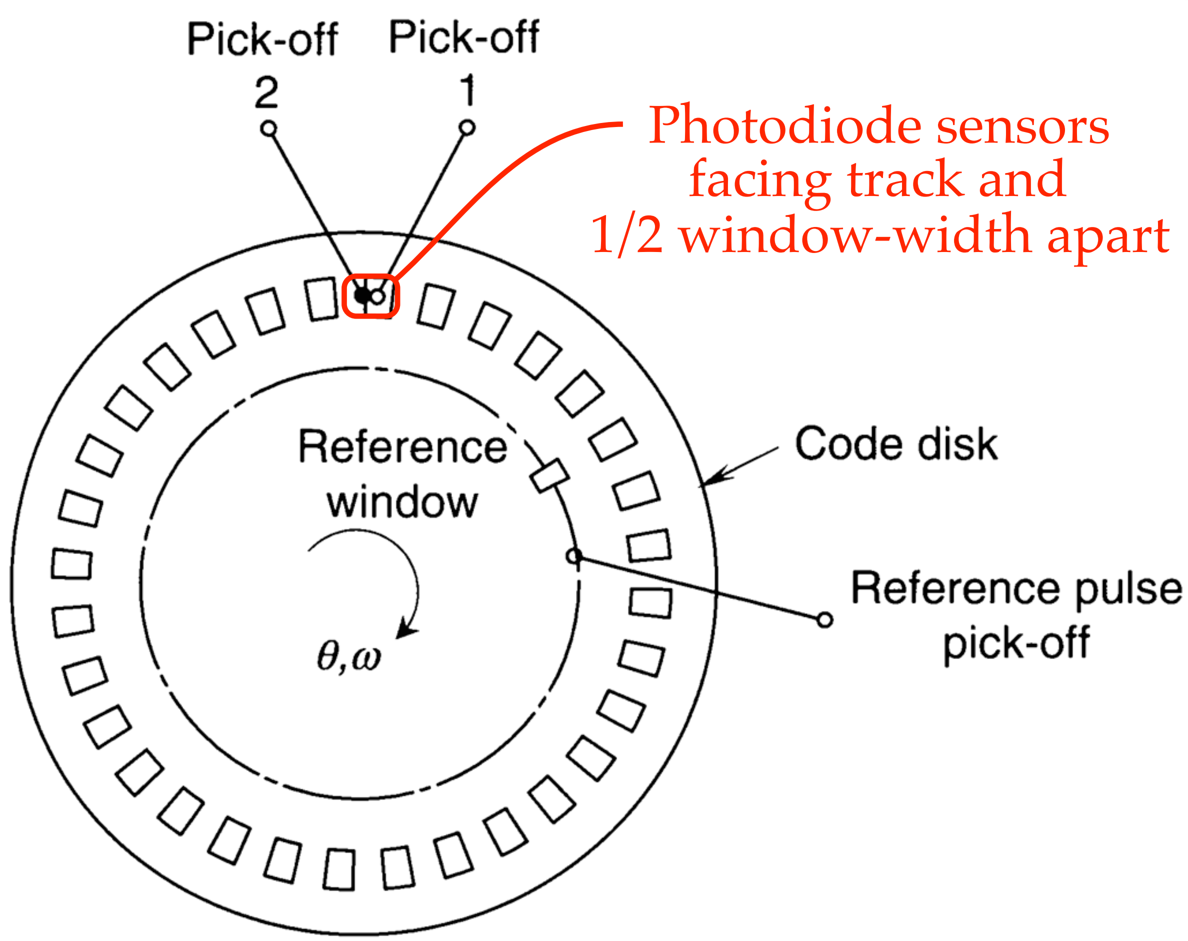


Under counter-clockwise rotation, v_1 goes high first, then, 1/4 cycle (90°) later, v_2 goes high.

Figure 6.4
Shaped pulse signals from an incremental encoder (a) for clockwise rotation; (b) for counterclockwise rotation; (c) reference pulse signal.

v_1 rising edge when v_2 is high $\Rightarrow$ clockwise



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An incremental encoder disk (offset sensor configuration).

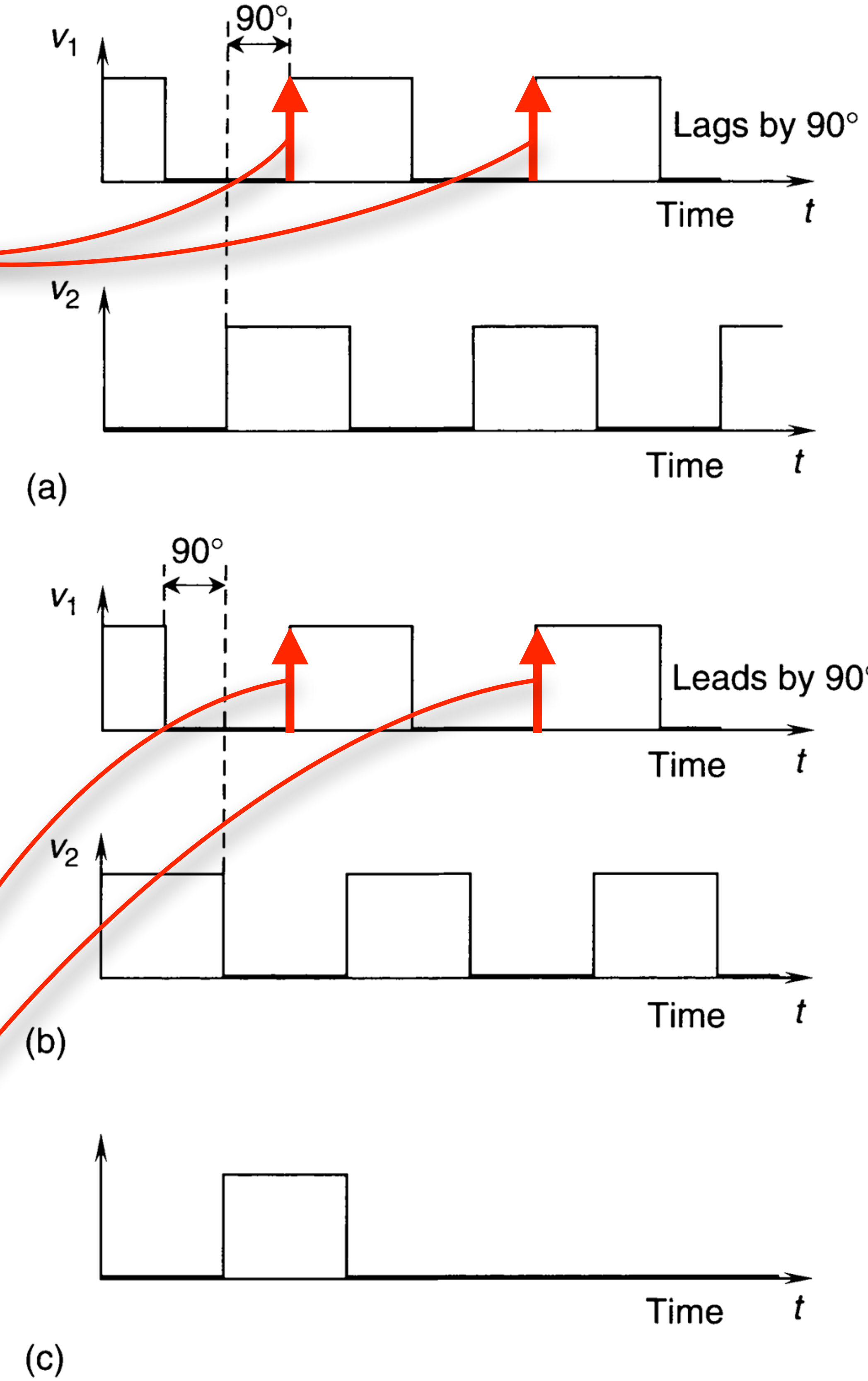


Figure 6.4
Shaped pulse signals from an incremental encoder (a) for clockwise rotation; (b) for counterclockwise rotation; (c) reference pulse signal.

v_1 rising edge when v_2 is low $\Rightarrow$ counter-clockwise

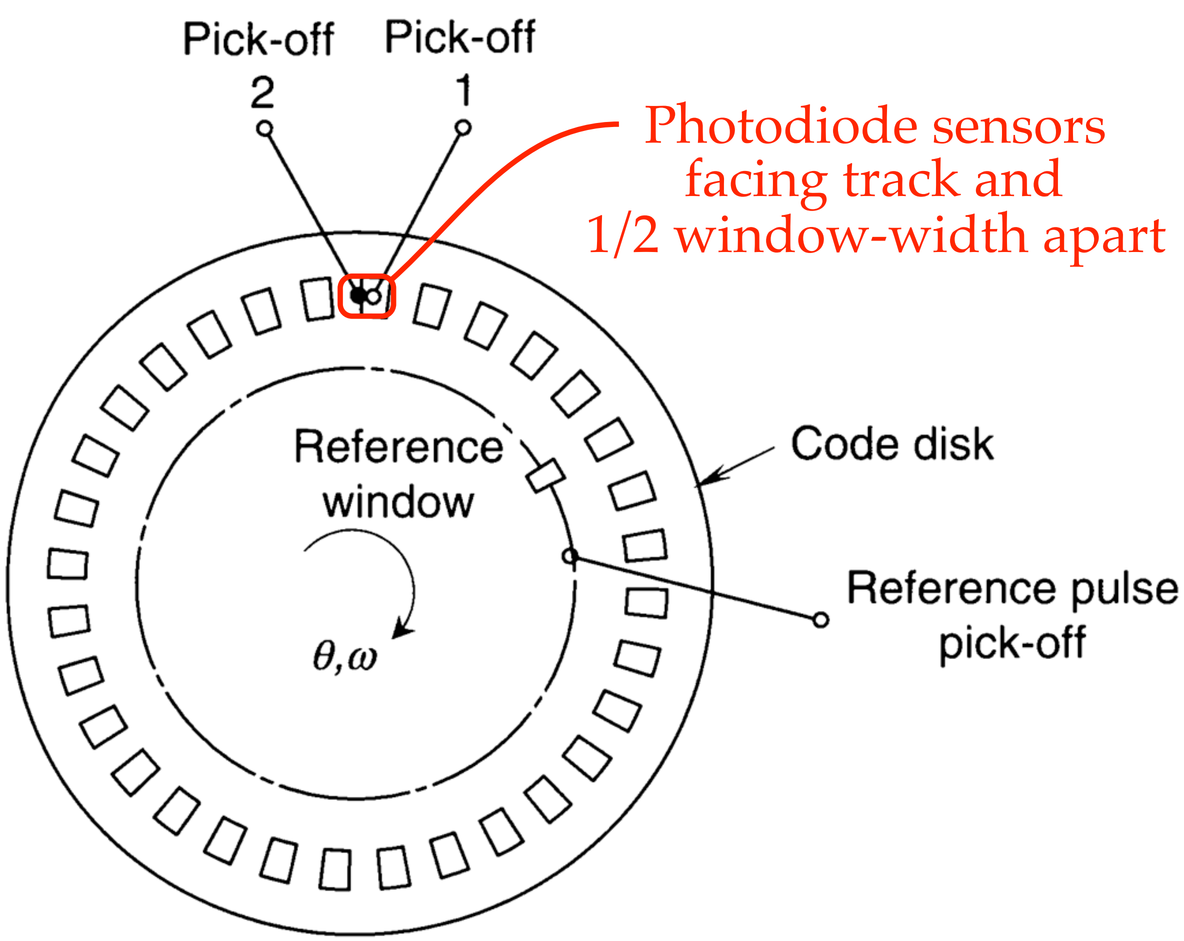
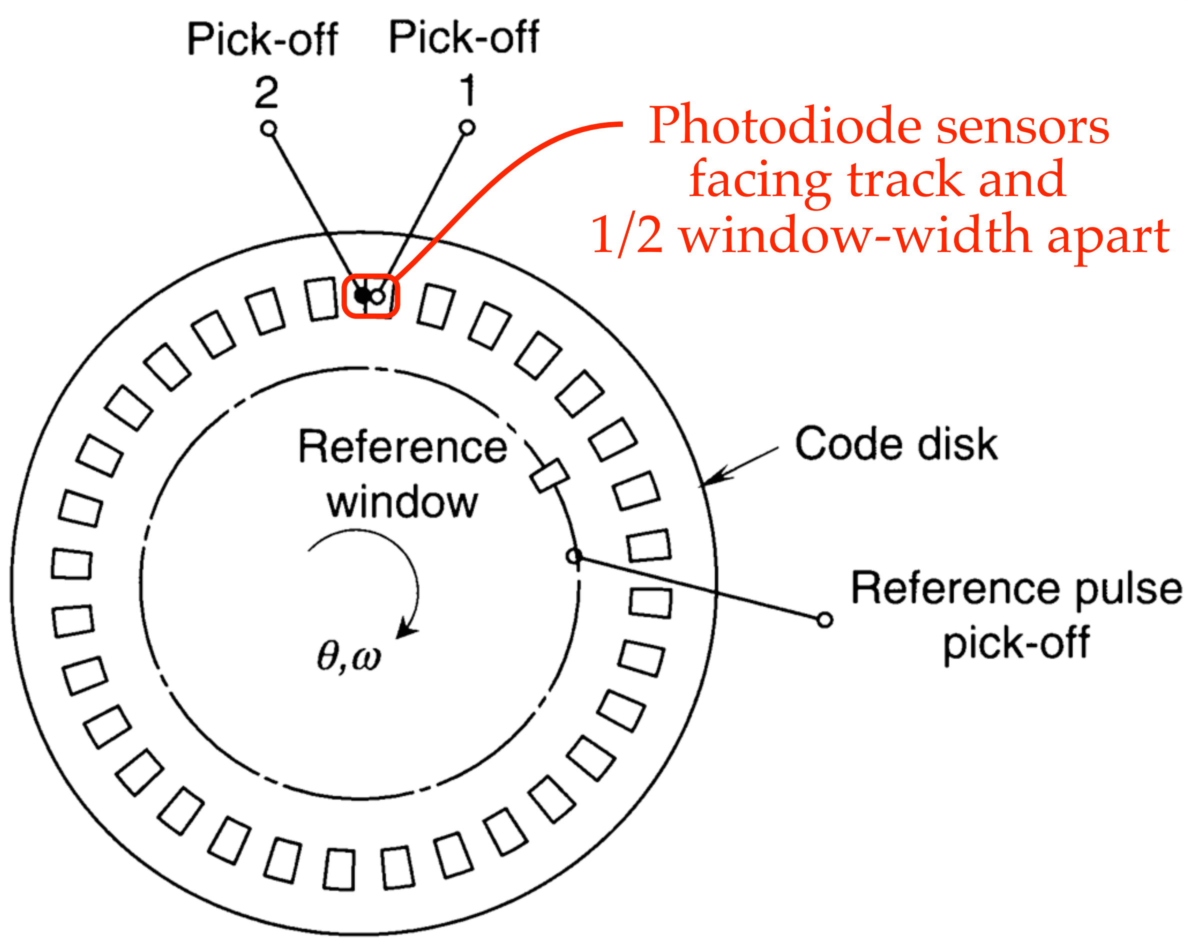


Figure 6.3
An incremental encoder disk (offset sensor configuration).

Another configuration: Two photodiode sensors, not offset, but with two tracks; one track offset 1/4 pitch w.r.t. the other.

v_1 rising edge when v_2 is high $\Rightarrow$ clockwise



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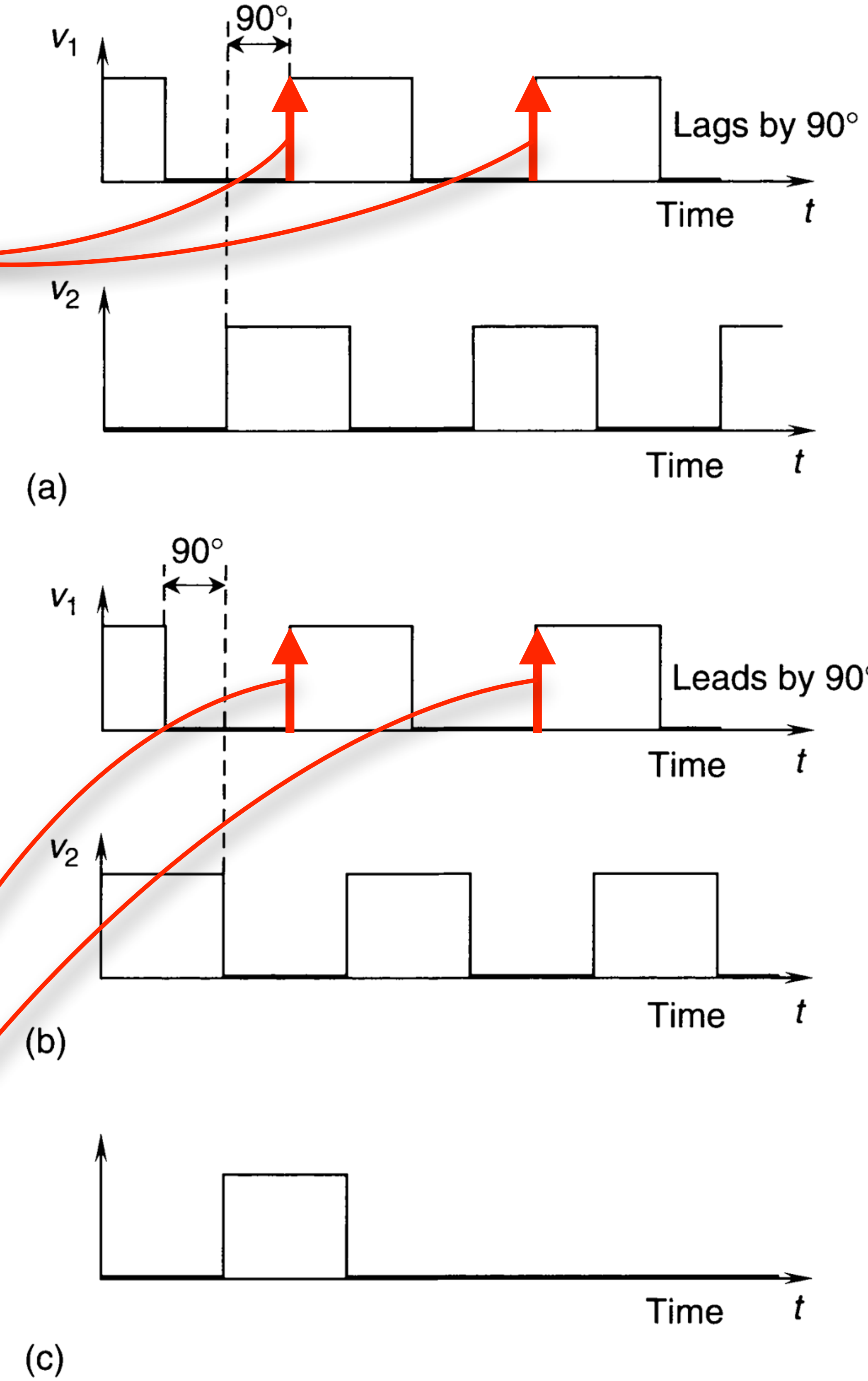

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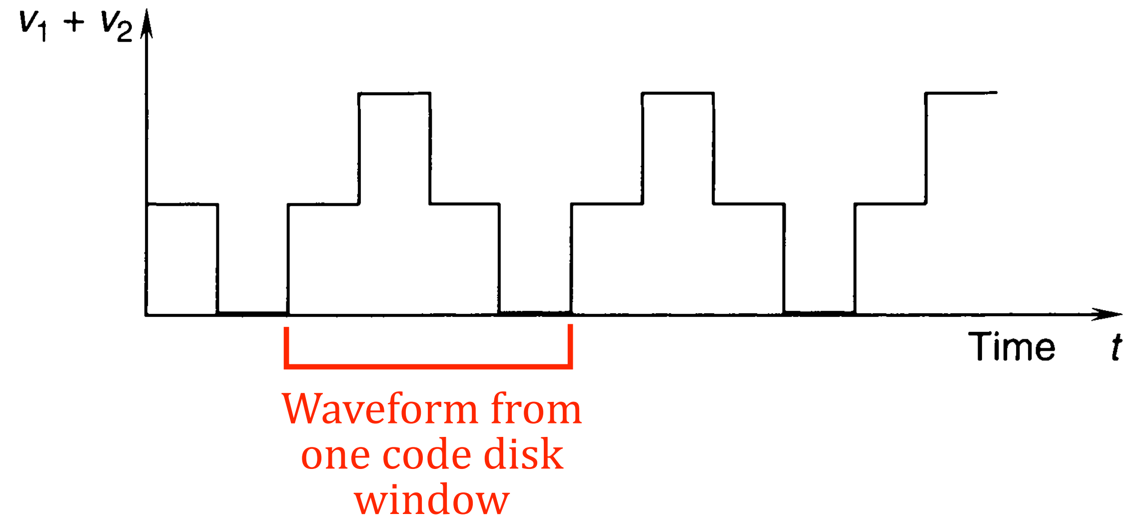

Figure 6.7
Quadrature signal addition to improve physical resolution.

6.4.1.2 Physical Resolution

The physical resolution of an encoder is governed by the number of windows N in the code disk. If only one pulse signal is used (i.e., no direction sensing) and if only the rising edges of the pulses are detected (i.e., full cycles of the encoder signal are counted), the physical resolution is given by the pitch angle of the track (i.e., angular separation between adjacent windows), which is $(360/N)°$. However, when quadrature signals (i.e., two pulse signals, one out of phase with the other by 90° or quarter-of-a-pitch angle) are available and the capability to detect both rising and falling edges of a pulse is also present, four counts can be made per encoder cycle, thereby improving the resolution by a factor of four. Under these conditions, the physical resolution of an encoder is given by

$$\Delta\theta_p = \frac{360°}{4N} \tag{6.8}$$

To understand this, note in Figure 6.4a (or Figure 6.4b) that when the two signals v_1 and v_2 are added, the resulting signal has a transition at every quarter of the encoder cycle. This is illustrated in Figure 6.7. By detecting each transition (through edge detection or level detection), four pulses can be counted within every main cycle. It should be mentioned that each signal (v_1 or v_2) has a resolution of half a pitch separately, provided that all transitions (rising edges and falling edges) are detected and counted instead of counting pulses (or high signal levels).

6.4.1.1 Digital Resolution

The resolution of an encoder represents the smallest change in measurement that can be measured realistically. Since an encoder can be used to measure both displacement and velocity, we can identify a resolution for each case. Here, we consider the displacement resolution, which is governed by the number of windows N in the code disk and the digital size (number of bits) of the buffer or register where the counter output is stored. Now we discuss digital resolution.

The displacement resolution of an incremental encoder is given by the change in displacement corresponding to a unit change in the count (n). It follows from Equation 6.1 that the displacement resolution is given by

$$\Delta\theta = \frac{\theta_{\max}}{M} \tag{6.2}$$

The digital resolution corresponds to a unit change in the bit value. Suppose that the encoder count is stored as digital data of r bits. Allowing for a sign bit, we have $M = 2^{r-1}$. By substituting this into Equation 6.2, we have the digital resolution

$$\Delta\theta_d = \frac{\theta_{\max}}{2^{r-1}} \tag{6.3}$$

Typically, $\theta_{\max} = \pm 180°$ or 360°. Then,

$$\boxed{\Delta\theta_d = \frac{180°}{2^{r-1}} = \frac{360°}{2^r}} \tag{6.4}$$

Note: The minimum count corresponds to the case where all the bits are zero and the maximum count corresponds to the case where all the bits are unity. Suppose that these two readings represent the angular displacements $\theta_{\min}$ and $\theta_{\max}$. We have

$$\theta_{\max} = \theta_{\min} + (M-1)\Delta\theta \tag{6.5}$$

or substituting $M = 2^{r-1}$ we have $\theta_{\max} = \theta_{\min} + (2^{r-1} - 1)\Delta\theta_d$. This gives the conventional definition for digital resolution:

$$\Delta\theta_d = \frac{(\theta_{\max} - \theta_{\min})}{(2^{r-1} - 1)} \tag{6.6}$$

Optical Encoder Analysis Example

The first joint of a robotic arm is a revolute ("rotational") joint that has a 300 degree range of motion. An incremental optical encoder is used to measure both the angular position and the angular velocity of the joint. The encoder's code disk has 900 windows (evenly spaced around the entire circumference of the disk). The encoder's two photodiode sensors deliver two pulse signals, one out-of-phase with the other by a quarter of a pitch cycle. The pulse signals from the two photodiode sensors are counted, using quadrature-decoding, counting up when the joint rotates clockwise, and the resulting count is stored in a 12-bit register.

Initially, the joint is rotated counter-clockwise to its limit ($\theta = 0$) position, where the 12-bit counter is zeroed.

(a) At a later time t_1, the count in the 12-bit counter is 0001 0000 0001. What is the measured angular position at this time? What is the resolution of this measurement?

(a) Quadrature decoding is being used, which betters the encoder resolution by a factor of 4 (compared to when only whole pulses are counted). So the resolution due to the encoder is

$$\Delta\theta_{encoder} = \frac{360 \text{ deg}}{4 \times N} = \frac{360 \text{ deg}}{4 \times 900} = 0.1 \text{ deg}$$

The resolution due to the counter is

$$\Delta\theta_{counter} = \frac{\theta_{\text{max}} - \theta_{\text{min}}}{2^r} = \frac{300^o - 0^o}{2^{12}} \approx 0.0732 \text{ deg}$$

The *measurement* resolution is the larger (the worst of) the two, which is 0.1 deg.

The count in the 12-bit counter is $2^8 + 2^0 = 257$. So the measured position is

$$\theta_{measured} = 257 \times 0.1 \text{ deg} = 25.7 \text{ deg}$$

Count		Joint Angle
0000 0000 0000	$=$	$0 \times 0.1 \text{ deg} = 0.0 \text{ deg}$
0000 0000 0001	$=$	$2^0 \times 0.1 \text{ deg} = 0.1 \text{ deg}$
0000 0000 0010	$=$	$2^1 \times 0.1 \text{ deg} = 0.2 \text{ deg}$
0000 0000 0011	$=$	$(2^1 + 2^0) \times 0.1 \text{ deg} = 0.3 \text{ deg}$
$\vdots$	$\vdots$	$\vdots$
0001 0000 0001	$=$	$(2^8 + 2^0) \times 0.1 \text{ deg} = 25.7 \text{ deg}$
$\vdots$	$\vdots$	$\vdots$
1011 1011 1000	$=$	$(2^{11} + 2^9 + 2^8 + 2^7 + 2^5 + 2^4 + 2^3) \times 0.1 \text{ deg} = 300.0 \text{ deg}$

Angular Speed Measurement with Encoders

Suppose there are N windows on the code disk and quadrature decoding is not used.

Approach #1

Count the number, n, of pulses during an interval of length T sec.

$$\text{Angular Speed} = \frac{n \text{ pulses}}{T \text{ sec}}$$

Measurement updates every T sec

At measurement update, measurement is the average speed during the preceding T sec.

Average measurement latency = $T/2$ sec

Approach #2

Count the number, m, of intervals of length T sec between adjacent (in time) pulses.

Computerized version of *measure the time between adjacent-in-time pulses.*

Angular Speed Measurement with Encoders

Suppose there are N windows on the code disk and quadrature decoding is not used.

Approach #1

Count the number, n, of pulses during an interval of length T sec.

$$\text{Angular Speed} = \frac{n \text{ pulses}}{T \text{ sec}}$$

Measurement updates every T sec

At measurement update, measurement is the average speed during the preceding T sec.

Average measurement latency = $T/2$ sec

Approach #2

Count the number, m, of intervals of length T sec between adjacent (in time) pulses.

$$\text{Angular Speed} = \frac{1 \text{ pulse}}{mT \text{ sec}}$$

Measurement updates when pulse is detected

At measurement update, measurement is the average speed during the preceding preceding mT sec.

Measurement latency is speed-dependent

Angular Speed Measurement with Encoders

Suppose there are N windows on the code disk and quadrature decoding is not used.

Approach #1

Count the number, n, of pulses during an interval of length T sec.

$$\text{Angular Speed} = \frac{n\ \cancel{\text{pulses}}}{T\ \text{sec}} \times \frac{1\ \cancel{\text{window}}}{\cancel{\text{pulse}}} \times \frac{2\pi\ \text{rad}}{N\ \cancel{\text{windows}}}$$

$$= \frac{2\pi n}{NT}\ \frac{\text{rad}}{\text{sec}}$$

Approach #2

Count the number, m, of intervals of length T sec between adjacent (in time) pulses.

$$\text{Angular Speed} = \frac{1\ \cancel{\text{pulse}}}{mT\ \text{sec}} \times \frac{1\ \cancel{\text{window}}}{\cancel{\text{pulse}}} \times \frac{2\pi\ \text{rad}}{N\ \cancel{\text{windows}}}$$

$$= \frac{2\pi}{NmT}\ \frac{\text{rad}}{\text{sec}}$$

$\dfrac{1\ \text{window}}{4\ \text{pulses}}$ if quadrature decoding is used

Resolution of Angular Speed Measurement with Encoders

$$\text{resolution} = \text{the smallest change that can be detected}$$

Approach #1

$$\begin{matrix}\text{Angular}\\ \text{Speed}\end{matrix} = \frac{2\pi n}{NT}\,\frac{\text{rad}}{\text{sec}}$$

$$\text{Resolution} = \text{Angular Speed}(n) - \text{Angular Speed}(n-1)$$

$$= \frac{2\pi}{NT}\,\frac{\text{rad}}{\text{sec}}$$

Approach #2

$$\begin{matrix}\text{Angular}\\ \text{Speed}\end{matrix} = \frac{2\pi}{NmT}\,\frac{\text{rad}}{\text{sec}}$$

$$\text{Resolution} = \text{Angular Speed}(m) - \text{Angular Speed}(m+1)$$

$$= \frac{2\pi}{NmT}\,\frac{\text{rad}}{\text{sec}} - \frac{2\pi}{N(m+1)T}\,\frac{\text{rad}}{\text{sec}}$$

$$= \frac{2\pi(\cancel{m}+1)}{Nm(m+1)T}\,\frac{\text{rad}}{\text{sec}} - \frac{2\pi\cancel{m}}{Nm(m+1)T}\,\frac{\text{rad}}{\text{sec}}$$

$$= \frac{2\pi}{Nm(m+1)T}\,\frac{\text{rad}}{\text{sec}}$$

Angular Speed Measurement with Encoders**

Summary

However:

$\uparrow T \Rightarrow \uparrow$ Measurement Latency

Range	Protocol	Angular Speed (rad/sec)	Resolution (rad/sec) (the smaller the better)	Comments
High Speeds	Count the number, n, of pulses generated during an interval of length T sec	$\frac{2\pi n}{NT}$	$\frac{2\pi}{NT}$	For best resolution choose T large. For given T, resolution is independent of speed.
Low Speeds	Count the number, m, of intervals of length T sec between adjacent (in time) pulses	$\frac{2\pi}{NmT}$	$\frac{2\pi}{Nm(m+1)T}$	For best resolution choose T small so that m will be large. For given T, resolution improves quadratically with decreasing speed.

** This table assumes that quadrature decoding is not used. The angular speed and resolution formulas when quadrature decoding is used are the same, except for an additional factor of 4 in their denominators.

Optical Encoder Analysis Example

The first joint of a robotic arm is a revolute ("rotational") joint that has a 300 degree range of motion. An incremental optical encoder is used to measure both the angular position and the angular velocity of the joint. The encoder's code disk has 900 windows (evenly spaced around the entire circumference of the disk). The encoder's two photodiode sensors deliver two pulse signals, one out-of-phase with the other by a quarter of a pitch cycle. The pulse signals from the two photodiode sensors are counted, using quadrature-decoding, counting up when the joint rotates clockwise, and the resulting count is stored in a 12-bit register. A separate 8-bit counter counts the number of intervals of length 0.001 sec between adjacent (in time) changes to the 12-bit count.

Initially, the joint is rotated counter-clockwise to its limit ($\theta = 0$) position, where the 12-bit counter is zeroed.

(a) At a later time t_1, the count in the 12-bit counter is 0001 0000 0001. What is the measured angular position at this time? What is the resolution of this measurement?

(b) At the same time t_1, the count in the 8-bit counter is 0100 0000. What is the measured angular speed at this time? What is the resolution of this measurement?

Angular Speed Measurement with Encoders

Summary

Range	Protocol	Angular Speed (rad/sec)	Resolution (rad/sec) (the smaller the better)	Comments
High Speeds	Count the number, n, of pulses generated during an interval of length T sec	$\frac{2\pi n}{NT}$	$\frac{2\pi}{NT}$	For best resolution choose T large. For given T, resolution is independent of speed.
Low Speeds	Count the number, m, of intervals of length T sec between adjacent (in time) pulses	$\frac{2\pi}{NmT}$	$\frac{2\pi}{Nm(m+1)T}$	For best resolution choose T small so that m will be large. For given T, resolution improves quadratically with decreasing speed.

(b) The angular speed is being measured by the interval counting method. By this method, with quadrature decoding:

$$\text{Angular Speed} = \frac{2\,\pi}{4\,N\,m\,T}$$

$$\text{Angular Speed Resolution} = \frac{2\,\pi}{4\,N\,m(m+1)T}$$

Here m is the count in the 8-bit counter, which is 0100 0000 $= 2^6 = 64$, so

$$\text{Angular Speed} = \frac{2\,\pi}{4\times900\times64\times0.001\text{ sec}}$$

$$= \frac{2\,\pi}{230.4}\frac{\text{rad}}{\text{sec}} = \frac{360}{230.4}\frac{\text{deg}}{\text{sec}} = 1.5625\frac{\text{deg}}{\text{sec}}$$

$$\text{Angular Speed Resolution} = \frac{2\,\pi}{4\times900\times64\times65\times0.001\text{ sec}}$$

$$= \frac{2\,\pi}{14{,}976}\frac{\text{rad}}{\text{sec}} = \frac{360}{14{,}976}\frac{\text{deg}}{\text{sec}} \approx 0.0240\frac{\text{deg}}{\text{sec}}$$

Absolute Encoders

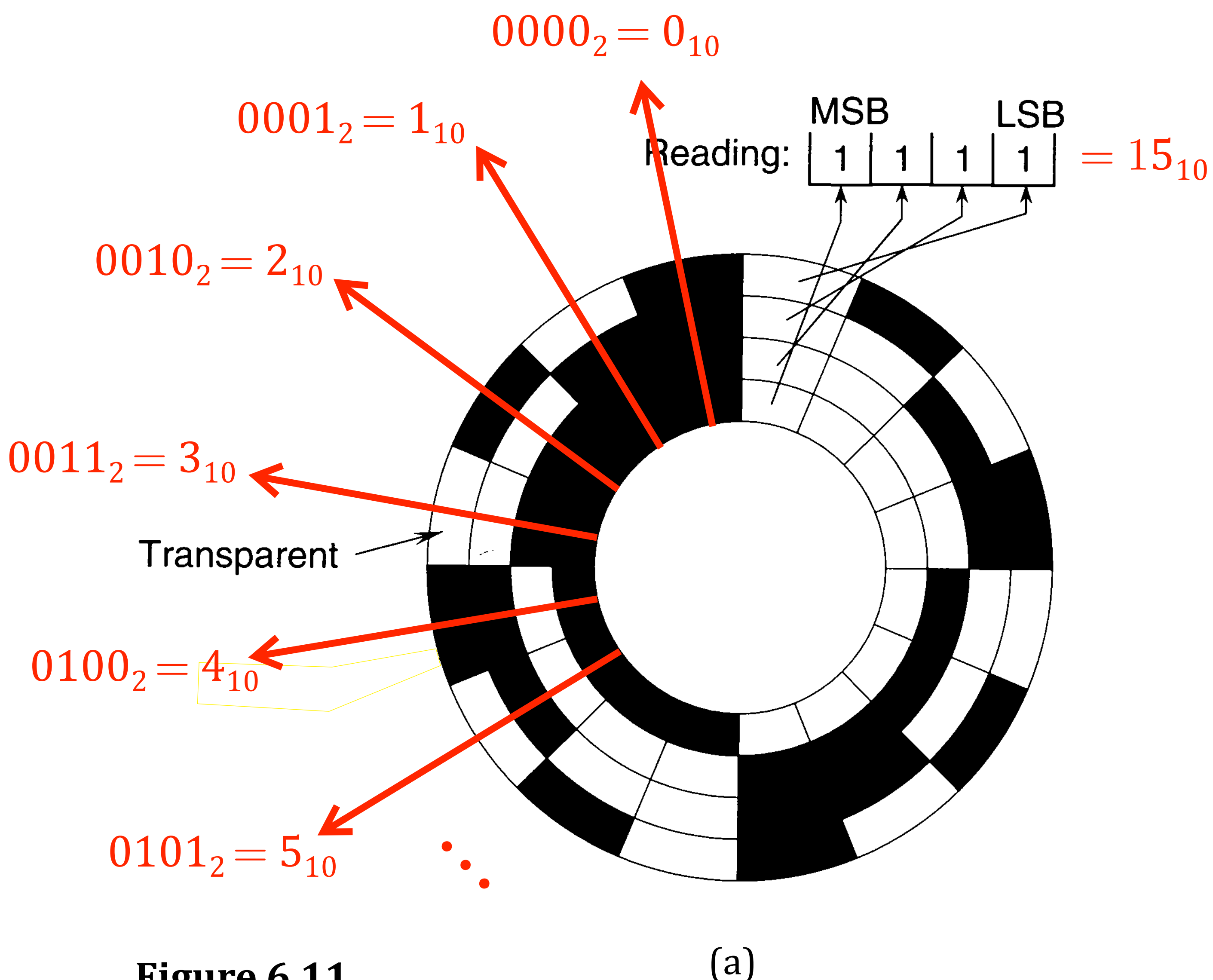

Figure 6.11
Illustration of the code pattern of an absolute encoder disk: (a) Binary code;

Table 6.2

Sector Coding for a 4-bit Absolute Encoder

Sector Number	Straight Binary Code (MSB → LSB)
0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
3	0 0 1 1
4	0 1 0 0
5	0 1 0 1
6	0 1 1 0
7	0 1 1 1
8	1 0 0 0
9	1 0 0 1
10	1 0 1 0
11	1 0 1 1
12	1 1 0 0
13	1 1 0 1
14	1 1 1 0
15	1 1 1 1

With the Straight Binary Code, during the transition from Sector 3 to Sector 4, unless all bits are converted absolutely synchronously, intermediate incorrect codes will appear.

Table 6.2

Sector Coding for a 4-bit Absolute Encoder

Sector Number	Straight Binary Code (MSB → LSB)	A Gray Code (MSB → LSB)
0	0 0 0 0	0 0 0 0
1	0 0 0 1	0 0 0 1
2	0 0 1 0	0 0 1 1
3	0 0 1 1	0 0 1 0
4	0 1 0 0	0 1 1 0
5	0 1 0 1	0 1 1 1
6	0 1 1 0	0 1 0 1
7	0 1 1 1	0 1 0 0
8	1 0 0 0	1 1 0 0
9	1 0 0 1	1 1 0 1
10	1 0 1 0	1 1 1 1
11	1 0 1 1	1 1 1 0
12	1 1 0 0	1 0 1 0
13	1 1 0 1	1 0 1 1
14	1 1 1 0	1 0 0 1
15	1 1 1 1	1 0 0 0

With the Straight Binary Code, during the transition from Sector 3 to Sector 4, unless all bits are converted absolutely synchronously, intermediate incorrect codes will appear.

Gray Code advantage: When the code disk rotates from one sector to the next, only one bit changes.