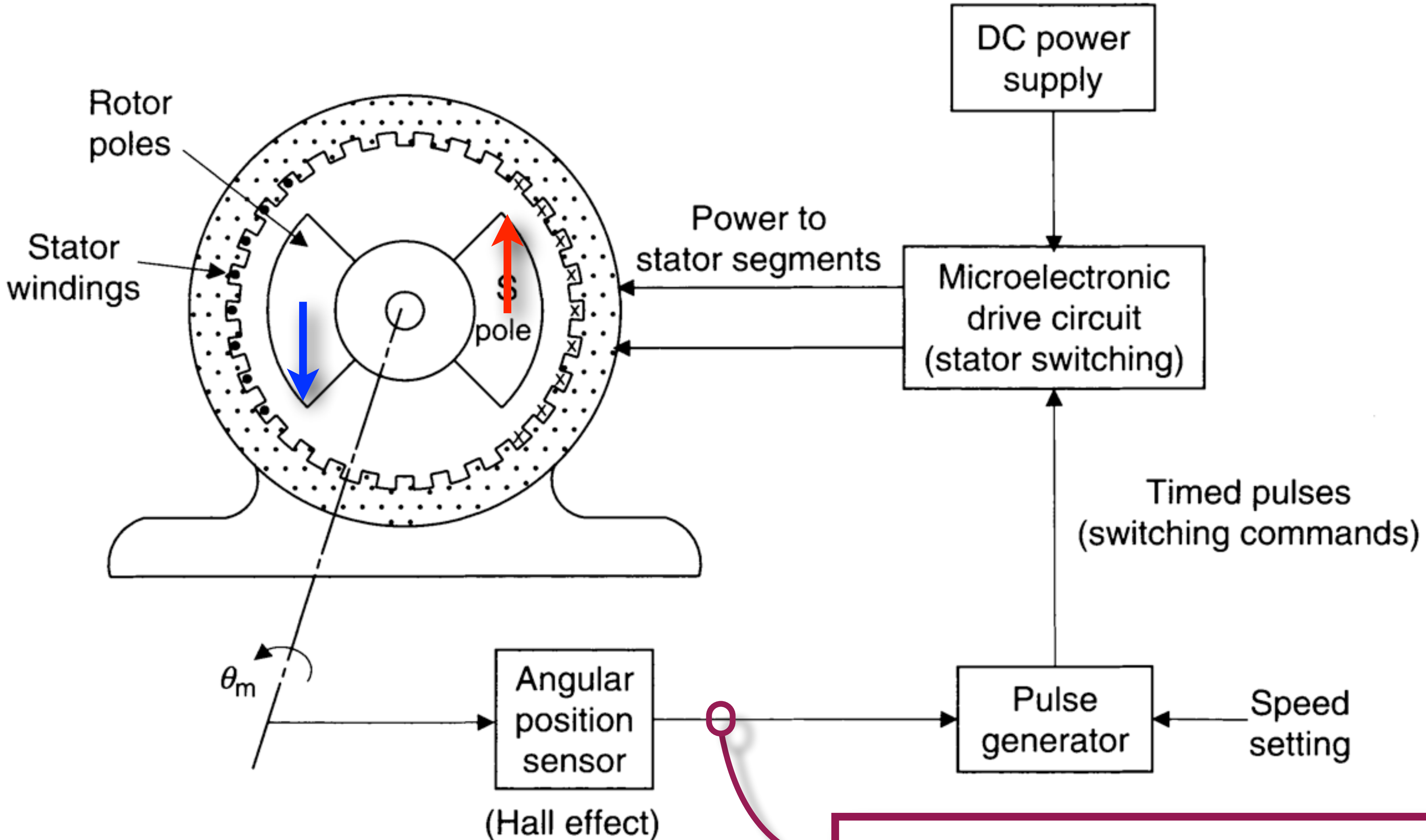


Figure 9.6
A brushless dc motor system.

Much in common with a stepper motor, except for measurement of rotor position.

Operating Principle

Switch the directions of the currents in the stator windings to rotate the magnetic field generated by the stator around the axis of rotation of the rotor. The rotor can then be a simple multiple-pole permanent magnet, but the directions of the currents in the stator windings must be switched in relation to the angular position, θ_m, of the rotor.

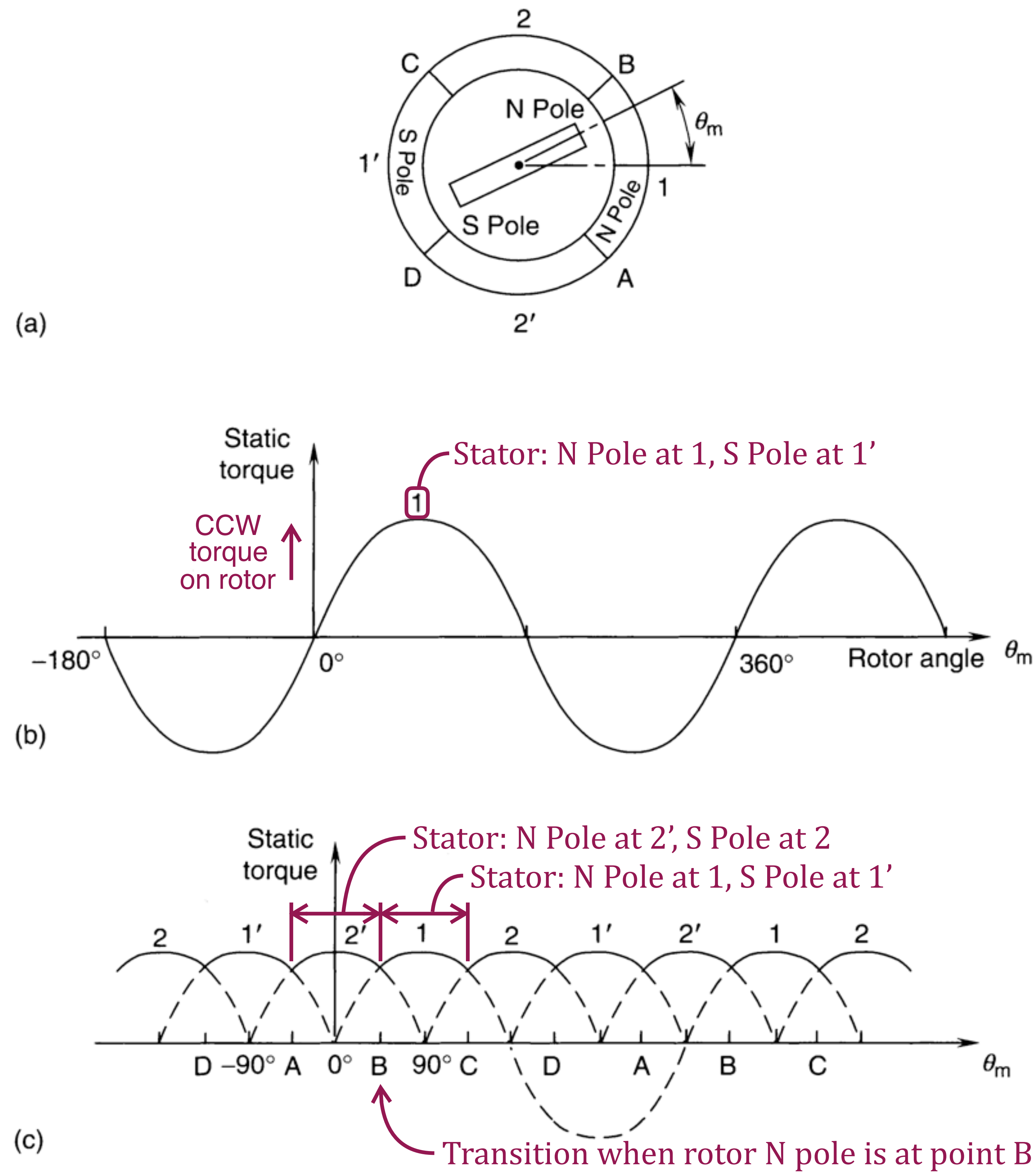


Figure 9.7

(a) A brushless dc motor. (b) Static torque curve with no switching (one-stator segment energized). (c) Switching sequence for maximum average torque.

DC Motor Dynamics

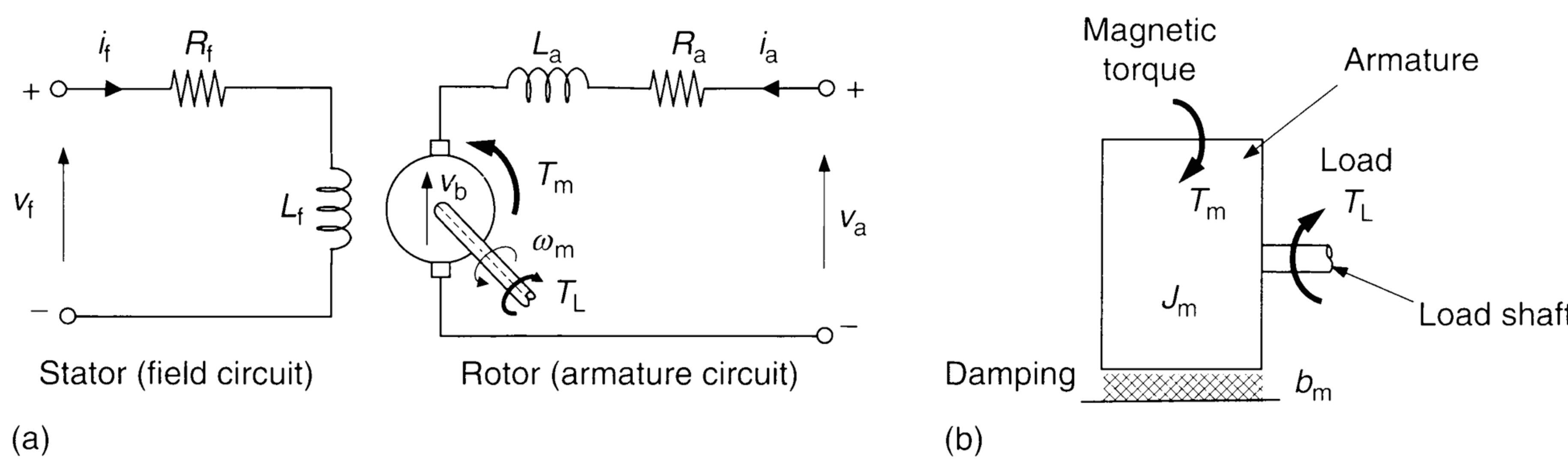


Figure 9.8
(a) The equivalent circuit of a conventional dc motor (separately excited). (b) Armature mechanical loading diagram.

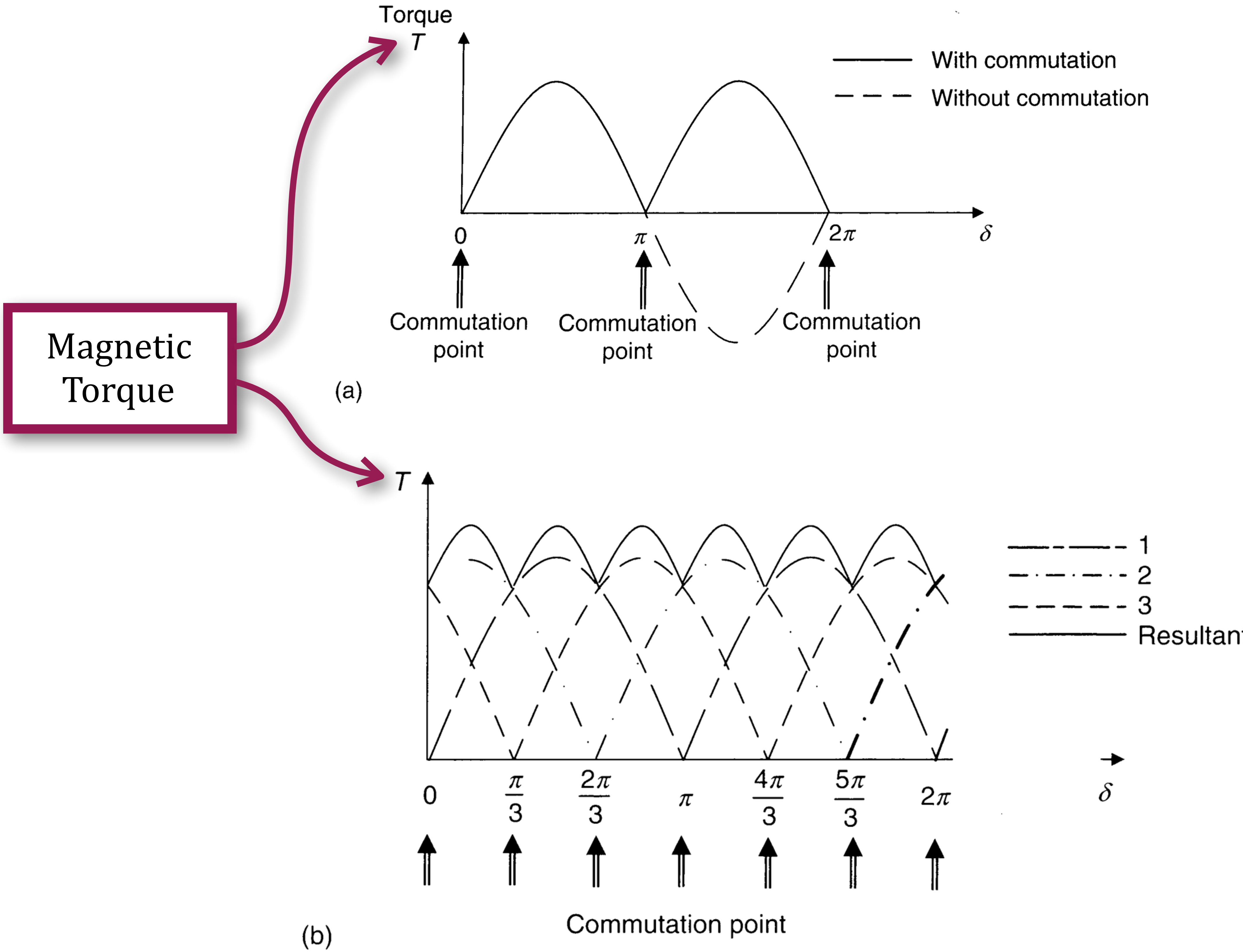


Figure 9.5

(a) Torque profile from a coil segment due to commutation.

(b) Resultant torque from a rotor with three-coil segments.

DC Motor Dynamics

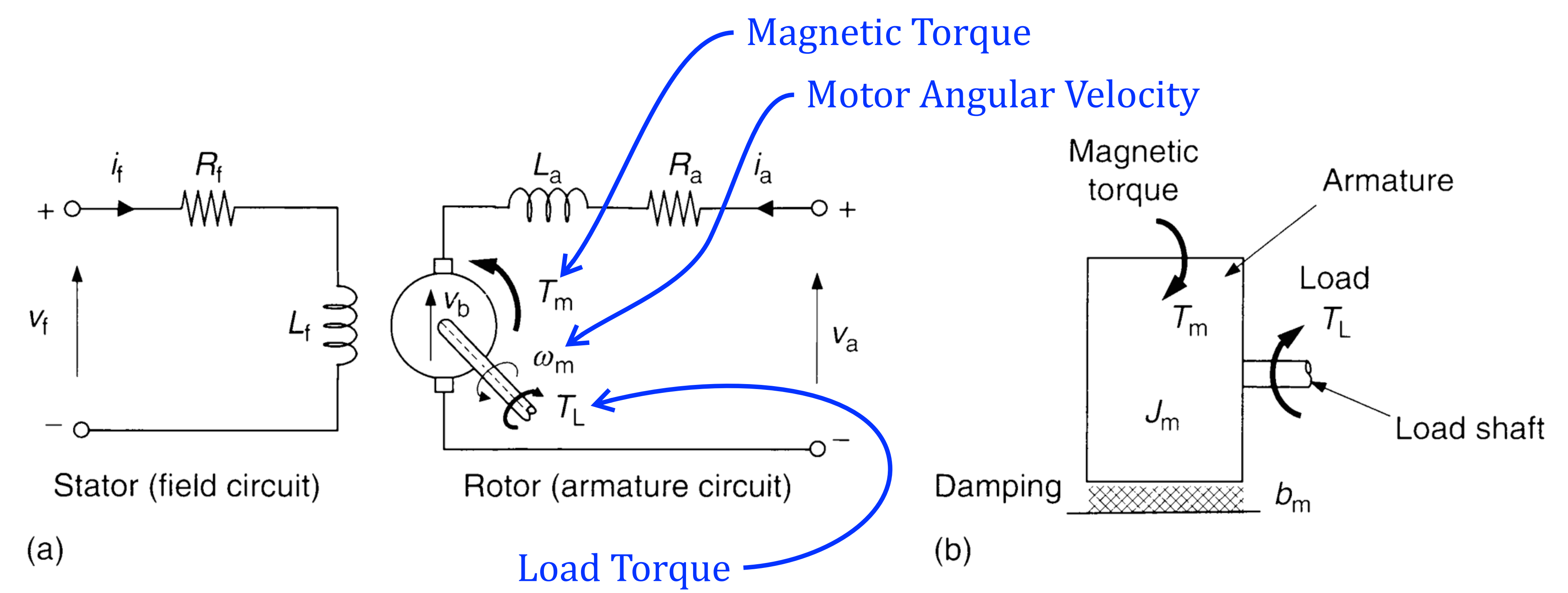
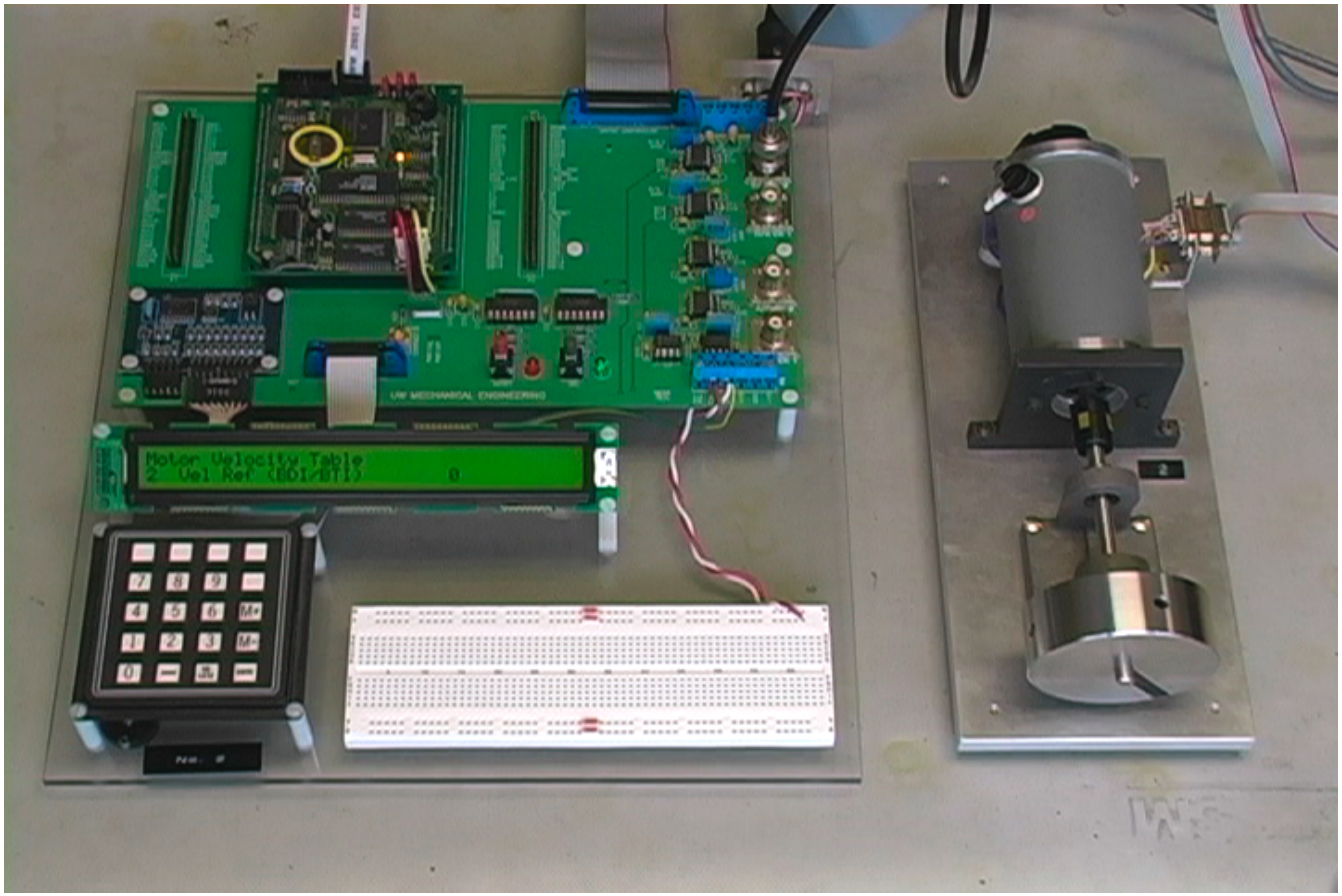


Figure 9.8
(a) The equivalent circuit of a conventional dc motor (separately excited). (b) Armature mechanical loading diagram.

Closed-Loop Control of DC Motor Angular Velocity



Start/End: Space Bar   Pause/Resume: K   Rewind: J   Fast-Forward: L   Jump to Beginning: I   Jump to End: O

Recall this earlier slide ...

$$\boldsymbol{F} = \text{force on conductor of length } l = \boldsymbol{i}\, l \times \boldsymbol{B}$$

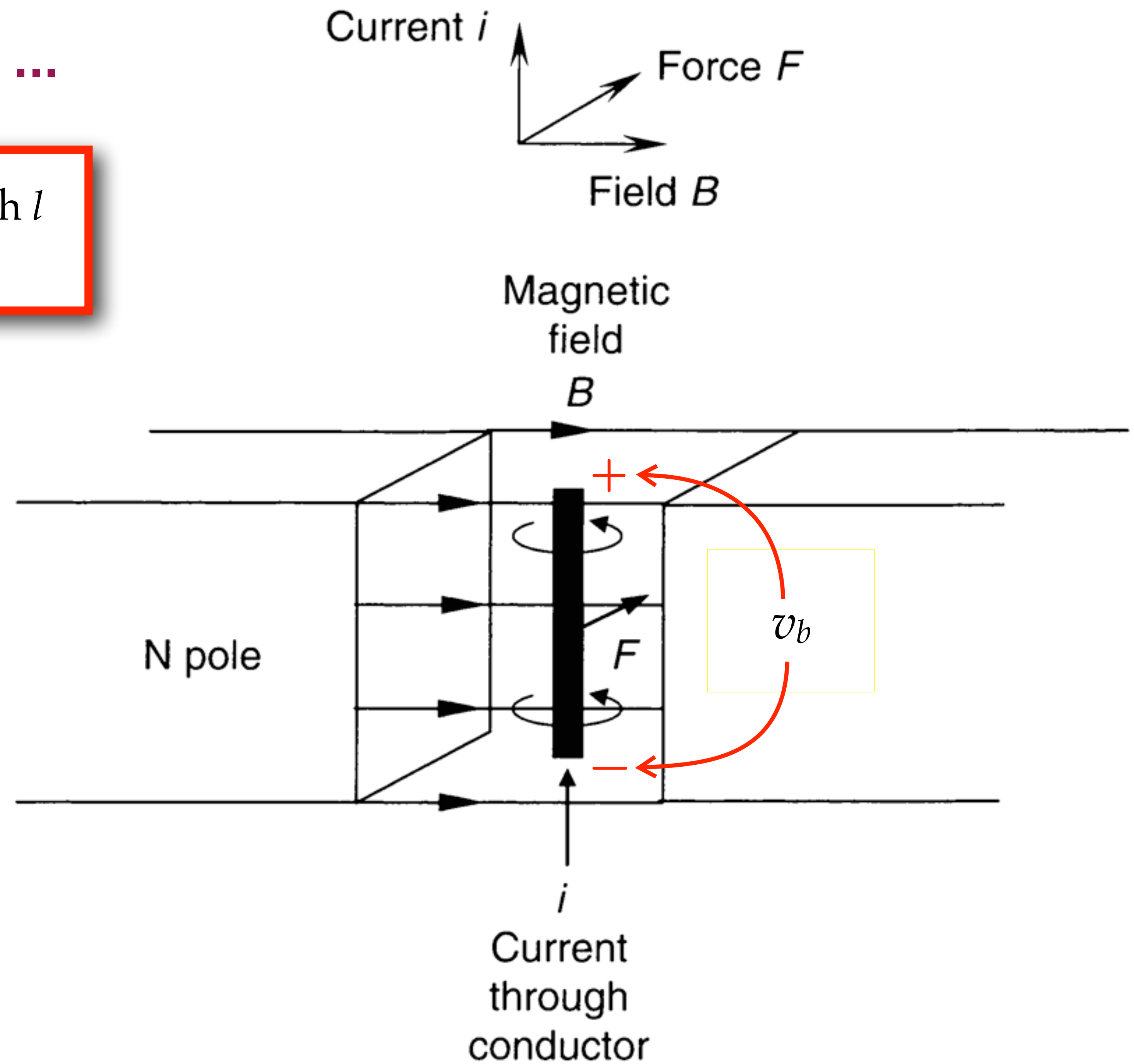


Figure 9.1
Operating principle of a dc motor.

If the conductor is free to move, then we can calculate the voltage across it:

$$v_b = \text{voltage induced across conductor due to its velocity } \boldsymbol{v} \text{ in the direction of } \boldsymbol{F}$$
$$= \text{the "back electromotive force"} = \text{the "back e.m.f."}$$
$$= B\, l\, v$$

9.3 DC Motor Equations

Equivalent circuits for the stator and the rotor of a conventional dc motor are shown in Figure 9.8a. Since the field flux is proportional to the field current i_f, we can express the magnetic torque of the motor as

$$T_m = k i_f i_a = k_m i_a \qquad \text{(magnetic torque)} \tag{9.4}$$

which directly follows Equation 9.1. Next, in view of Equation 9.2, the back e.m.f. generated in the armature of the motor is given by

$$v_b = k' i_f \omega_m = k'_m \omega_m \tag{9.5}$$

(“back electro-motive force voltage”)

where

i_f is the field current
i_a is the armature current
ω_m is the angular speed of the motor
k and k' are motor constants, which depend on factors such as the rotor dimensions, the number of turns in the armature windings, and the *permeability* (inverse of *reluctance*) of the magnetic medium

In the case of ideal electrical-to-mechanical energy conversion at the rotor (where the rotor coil links with the stator field), we have $T_m \times \omega_m = v_b \times i_a$, when consistent units are used (e.g., torque in newton-meters, speed in radians per second, voltage in volts, and current in amperes). Then we observe that

$$k = k' \quad \text{or} \quad k_m = k'_m \tag{9.6}$$

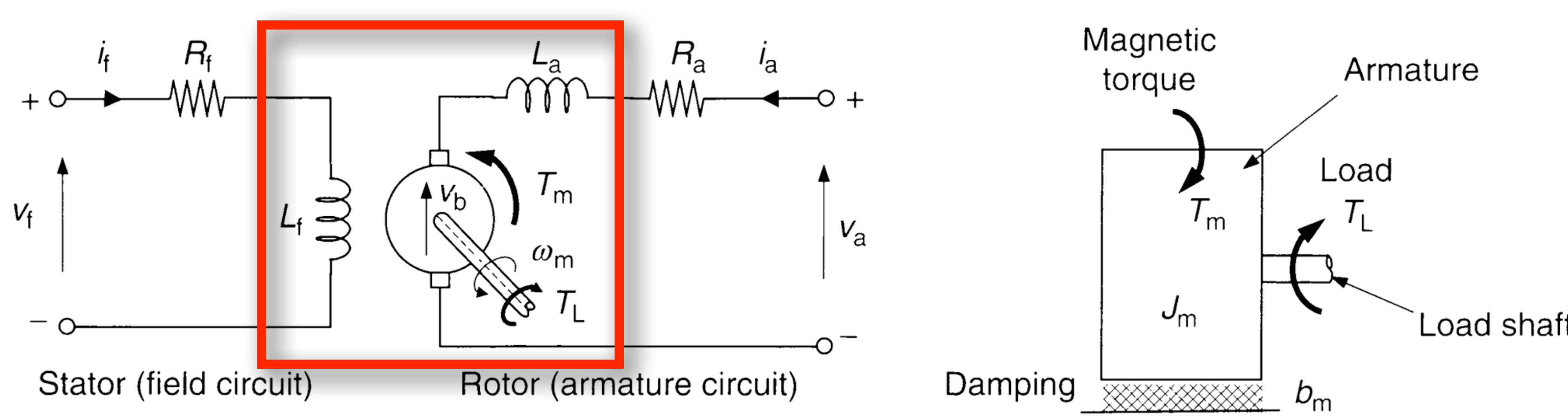


Figure 9.8 (a)

Figure 9.8 (b)

9.3 DC Motor Equations

Equivalent circuits for the stator and the rotor of a conventional dc motor are shown in Figure 9.8a. Since the field flux is proportional to the field current i_f, we can express the magnetic torque of the motor as

$$T_m = k i_f i_a = k_m i_a \tag{9.4}$$

which directly follows Equation 9.1. Next, in view of Equation 9.2, the back e.m.f. generated in the armature of the motor is given by

$$v_b = k' i_f \omega_m = k'_m \omega_m \tag{9.5}$$

under ideal energy conversion:

$$\left.\begin{array}{lll}\text{magnetic torque} & T_m = k\, i_f i_a & \\ \text{back e.m.f.} & v_b = k'\, i_f \omega_m & \end{array}\right\} \Rightarrow T_m\omega_m = (k\, i_f i_a)\left(\frac{v_b}{k'\, i_f}\right) = \frac{k}{k'} v_b\, i_a$$

$$\text{equate power: } \quad T_m \omega_m = v_b\, i_a \quad \Rightarrow k = k'$$

In the case of ideal electrical-to-mechanical energy conversion at the rotor (where the rotor coil links with the stator field), we have $T_m \times \omega_m = v_b \times i_a$, when consistent units are used (e.g., torque in newton-meters, speed in radians per second, voltage in volts, and current in amperes). Then we observe that

$$k = k' \quad \text{or} \quad k_m = k'_m \tag{9.6}$$

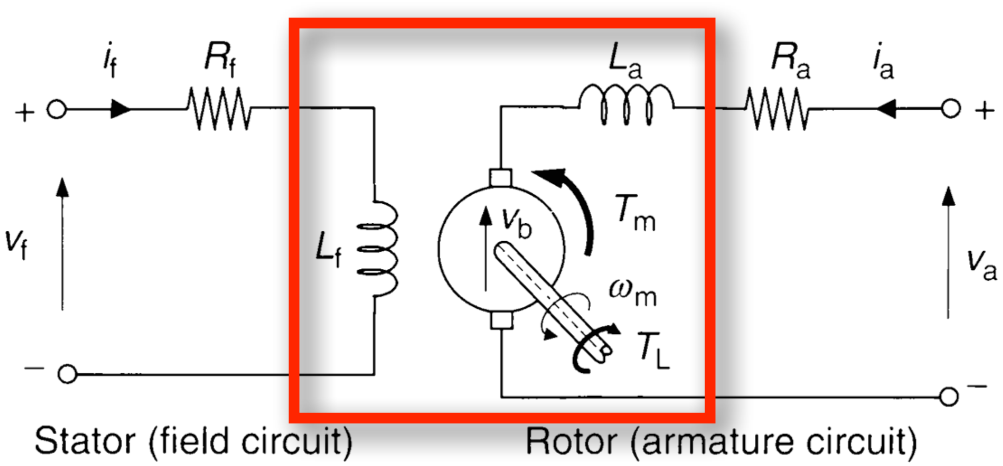


Figure 9.8 (a)

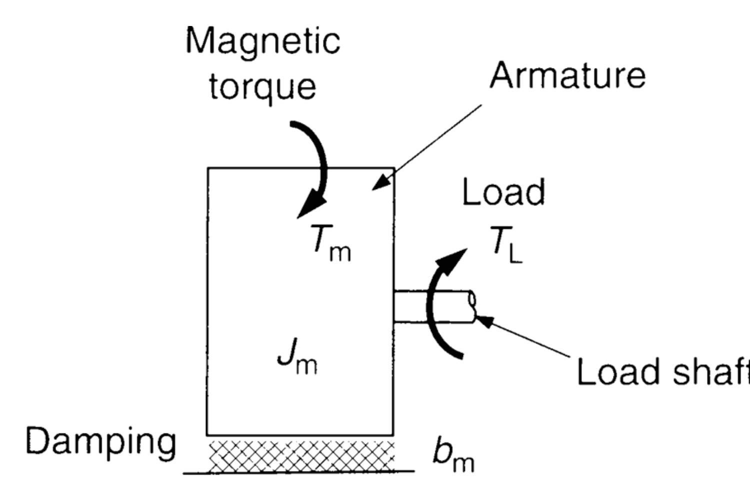


Figure 9.8 (b)

Field circuit: The field circuit equation is obtained by assuming that the stator magnetic field is not affected by the rotor magnetic field (i.e., the stator inductance is not affected by the rotor) and that there are no eddy current effects in the stator. Then, from Figure 9.8a,

$$v_f = R_f i_f + L_f \frac{di_f}{dt} \tag{9.7}$$

where

- v_f is the supply voltage to the stator
- R_f is the resistance of the field windings
- L_f is the inductance of the field windings

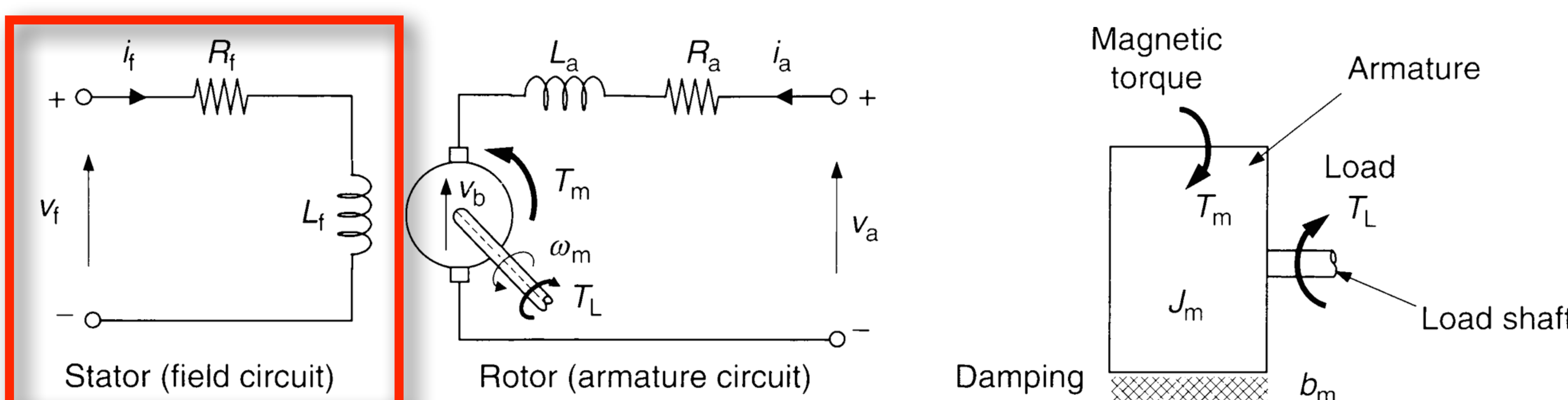


Figure 9.8 (a) **Figure 9.8** (b)

Armature circuit: The equation for the armature (rotor) circuit is written as (see Figure 9.8a)

$$v_a = R_a i_a + L_a \frac{di_a}{dt} + v_b \tag{9.8}$$

where

v_a is the supply voltage to the armature
R_a is the resistance of the armature windings
L_a is the leakage inductance in the armature windings

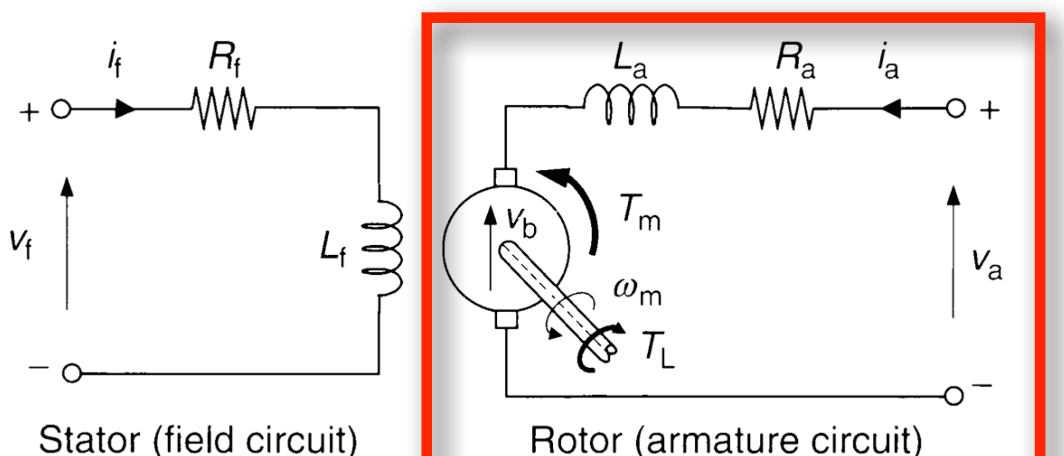


Figure 9.8 (a)

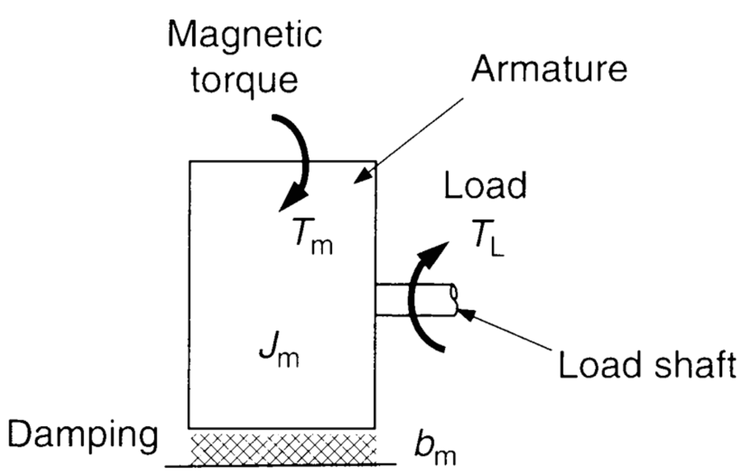


Figure 9.8 (b)

Mechanical dynamics: The mechanical equation of the motor is obtained by applying Newton's second law to the rotor. Assuming that the motor drives some load, which requires a load torque T_L to operate, and that the frictional resistance in the armature (e.g., in the bearings) can be modeled by a linear viscous term, we have (see Figure 9.8b)

$$J_m \frac{d\omega_m}{dt} = T_m - T_L - b_m\omega_m \tag{9.9}$$

where

J_m is the moment of inertia of the rotor
b_m is the equivalent mechanical damping constant for the rotor

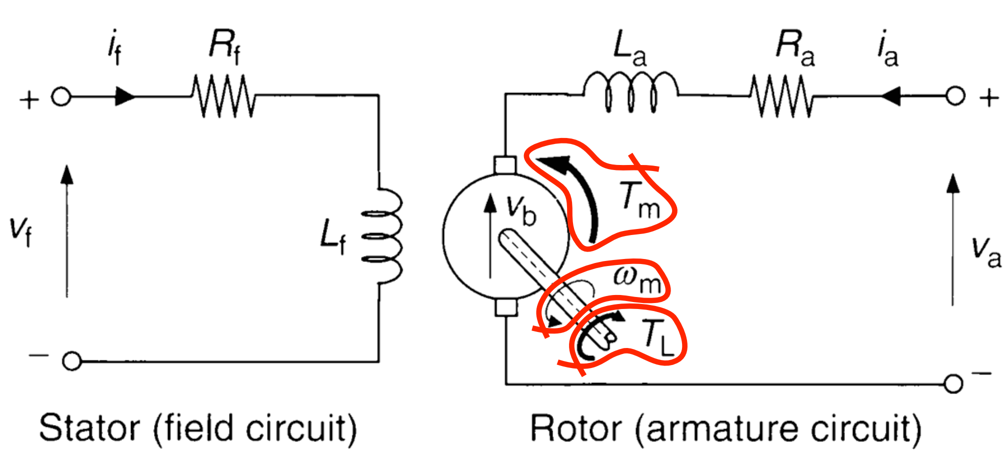


Figure 9.8 (a)

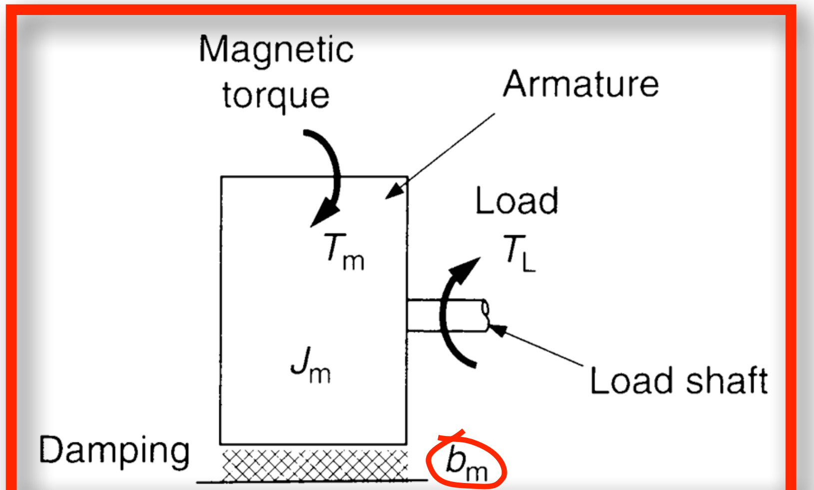


Figure 9.8 (b)

DC Motor Equations:

$$T_m = k i_f i_a \tag{9.4}$$

$$v_b = k' i_f \omega_m \tag{9.5}$$

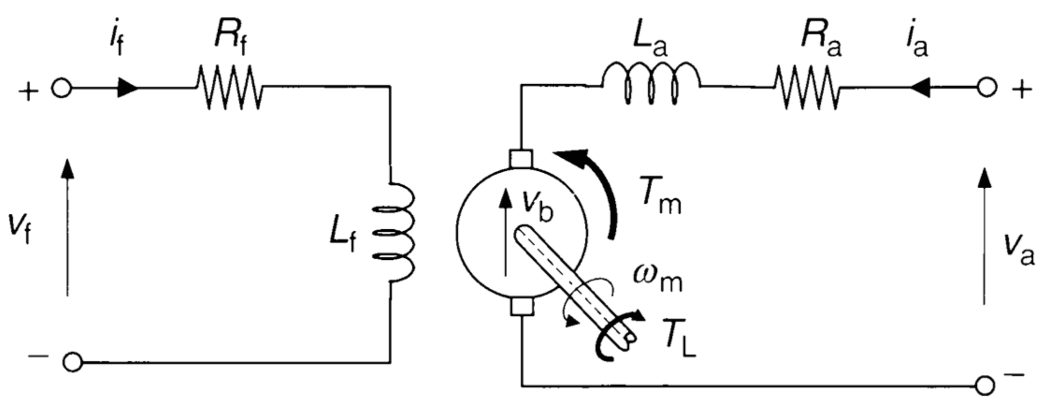
$$k = k' \tag{9.6}$$

$$v_f = R_f i_f + L_f \frac{di_f}{dt} \tag{9.7}$$

$$v_a = R_a i_a + L_a \frac{di_a}{dt} + v_b \tag{9.8}$$

$$J_m \frac{d\omega_m}{dt} = T_m - T_L - b_m \omega_m \tag{9.9}$$

Steady-State Speed-Torque Characteristics



With:

- v_f constant
- v_a constant
- T_L constant

in the steady state:

- ω_m will be constant
- T_m will be constant

What is the relationship between the constant v_f, v_a and T_L values and the resulting steady-state ω_m and T_m values?

Answer:

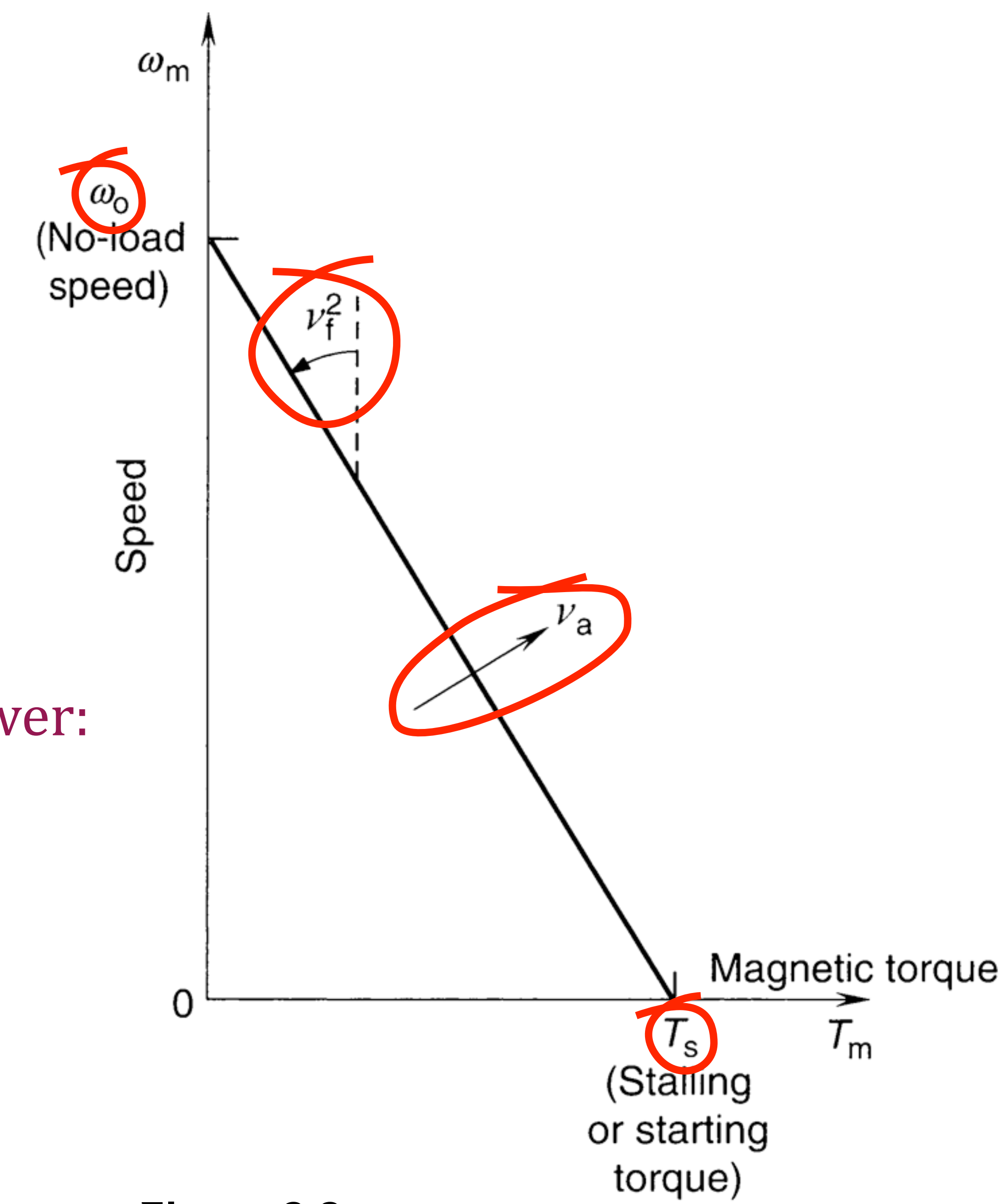
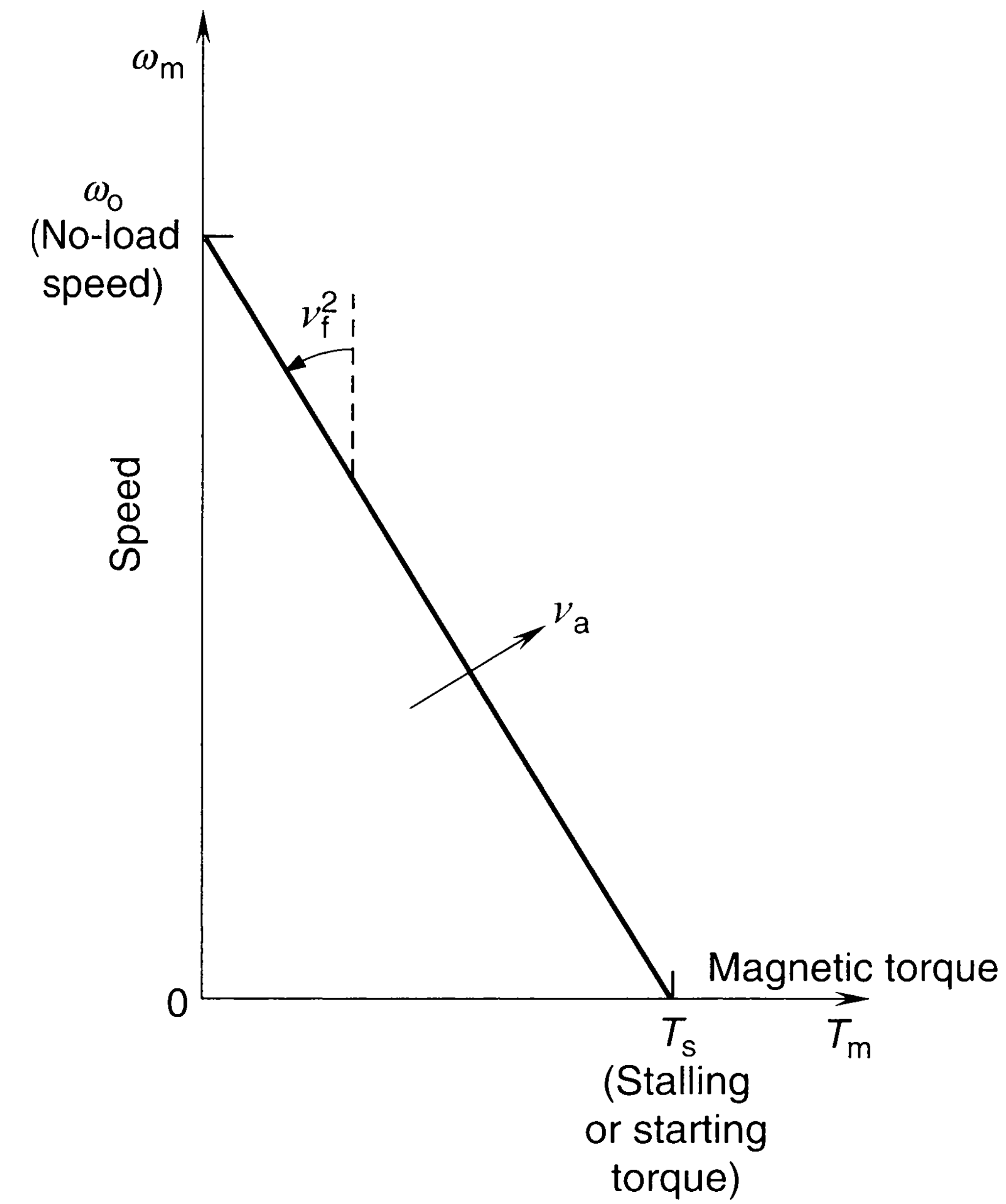


Figure 9.9
Steady-state speed–torque characteristics of a separately wound dc motor.

2015 Final Exam problem:



3. Figure 9.9, from our textbook, is shown above. Regarding that figure, explain, *in your own words*, the meaning of:

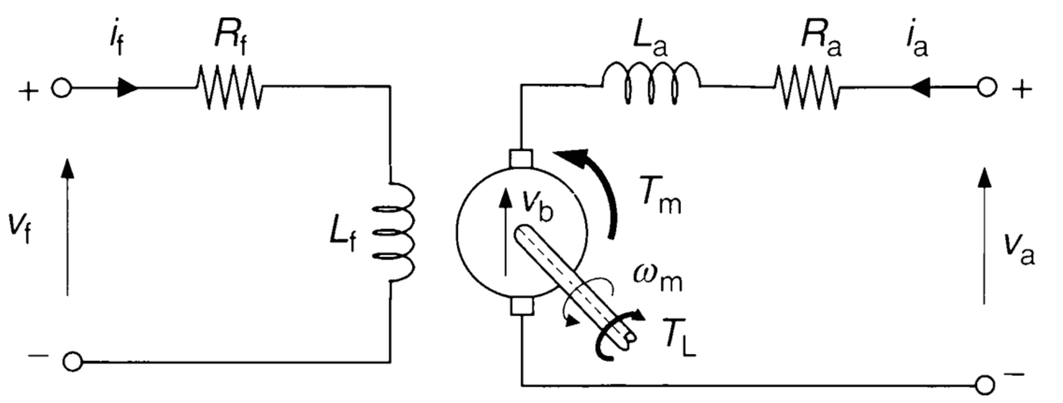
   (a) ω_o;

   (b) T_S;

   (c) the combination of the v_f^2 symbol and the counter-clockwise-pointing arc; and

   (d) the combination of the v_a symbol and the arrow.

Steady-State Speed-Torque Characteristics



With:

- v_f constant
- v_a constant
- T_L constant

in the steady state:

- ω_m will be constant
- T_m will be constant

What is the relationship between the constant v_f, v_a and T_L values and the resulting steady-state ω_m and T_m values?

Answer:

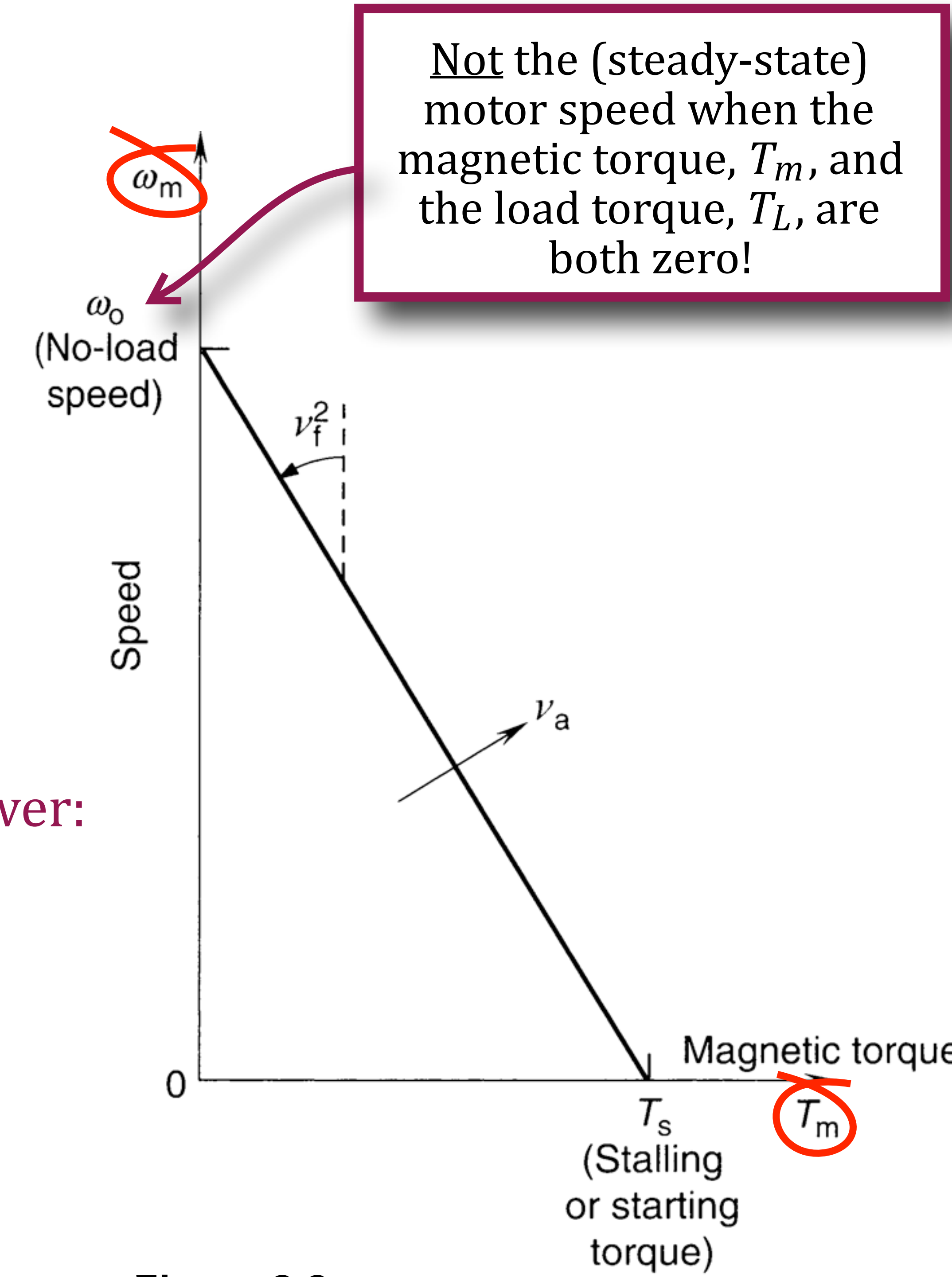
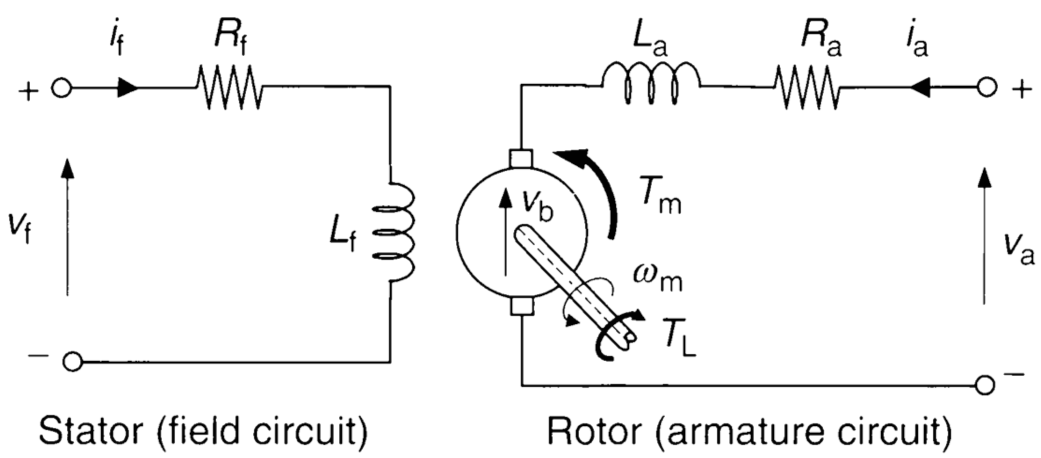


Figure 9.9
Steady-state speed–torque characteristics of a separately wound dc motor.

Steady-State Speed-Torque Characteristics



With:

- v_f constant
- v_a constant
- T_L constant

in the steady state:

- ω_m will be constant
- T_m will be constant

What is the relationship between the constant v_f, v_a and T_L values and the resulting steady-state ω_m and T_m values?

Answer:

The (steady-state) motor speed when the magnetic torque, T_m, is zero and the load torque, T_L, exactly counteracts the bearing friction torque $b_m \omega_0$.

Derivation Follows

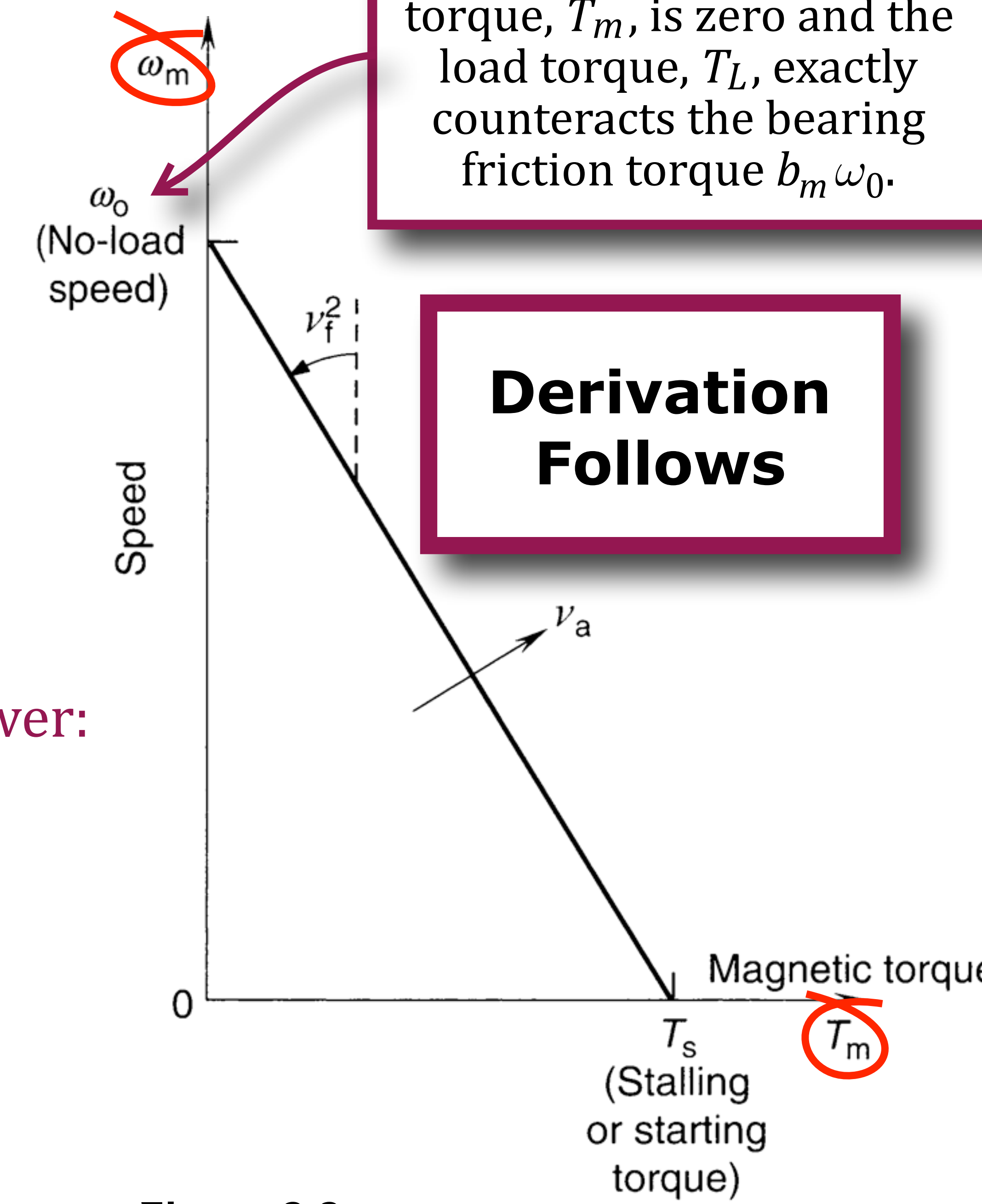


Figure 9.9
Steady-state speed–torque characteristics of a separately wound dc motor.

Derivation of Figure 9.9

Steady-State DC Motor Equations:

$$T_m = k i_f i_a \tag{9.4}$$

$$v_b = k' i_f \omega_m \tag{9.5}$$

$$k = k' \tag{9.6}$$

$$v_f = R_f i_f + L_f \cancelto{0}{\frac{di_f}{dt}} \tag{9.7}$$

$$v_a = R_a i_a + L_a \cancelto{0}{\frac{di_a}{dt}} + v_b \tag{9.8}$$

$$J_m \cancelto{0}{\frac{d\omega_m}{dt}} = T_m - T_L - b_m \omega_m \tag{9.9}$$

Derivation of Figure 9.9

Steady-State DC Motor Equations:

$$T_m = k i_f i_a \quad \Rightarrow i_a = \frac{T_m}{k i_f}$$

$$v_b = k' i_f \omega_m$$

$$k = k'$$

$$v_f = R_f i_f + L_f \cancelto{0}{\frac{di_f}{dt}} \quad \Rightarrow i_f = \frac{v_f}{R_f}$$

Multiply both sides by $\frac{R_f}{k' v_f}$

$$v_a = R_a i_a + L_a \cancelto{0}{\frac{di_a}{dt}} + v_b \quad \Rightarrow v_a = \frac{R_a}{k i_f} T_m + k' i_f \omega_m \quad \Rightarrow v_a = \frac{R_a R_f}{k\, v_f} T_m + k' \frac{v_f}{R_f} \omega_m$$

$$\Rightarrow \boxed{\frac{R_f v_a}{k' v_f} = \frac{R_a R_f^2}{k k' v_f^2} T_m + \omega_m}$$

When v_a, v_f and T_L are constant, in the steady state, this equation is satisfied.

The "stall torque" is the T_m that satisfies the above equation when $\omega_m = 0$.

The "no load speed" is the ω_m that satisfies the above equation when $T_m = 0$.

From equation 9.9, at the "no load speed", the load torque, T_L, cannot be zero! Instead, at the "no load speed", T_L must exactly counteract the bearing friction torque.

$$\boxed{J_m \cancelto{0}{\frac{d\omega_m}{dt}} = \cancelto{0}{T_m} - T_L - b_m \omega_m \qquad (9.9)}$$

Derivation of Figure 9.9

Steady-State DC Motor Equations:

$$T_m = k i_f i_a \quad \Rightarrow i_a = \frac{T_m}{k i_f}$$

$$v_b = k' i_f \omega_m$$

$$k = k'$$

$$v_f = R_f i_f + L_f \cancelto{0}{\frac{di_f}{dt}} \quad \Rightarrow i_f = \frac{v_f}{R_f}$$

$$v_a = R_a i_a + L_a \cancelto{0}{\frac{di_a}{dt}} + v_b \quad \Rightarrow v_a = \frac{R_a}{k i_f} T_m + k' i_f \omega_m \quad \Rightarrow v_a = \frac{R_a R_f}{k\, v_f} T_m + \overbrace{k' \frac{v_f}{R_f}}^{\text{Multiply both sides by } \frac{R_f}{k' v_f}} \omega_m$$

$$\Rightarrow \frac{R_f v_a}{k' v_f} = \frac{R_a R_f^2}{k k' v_f^2} T_m + \omega_m$$

And we have that:

The "stall torque", T_s, satisfies $\boxed{\frac{R_f v_a}{k' v_f} = \frac{R_a R_f^2}{k k' v_f^2} T_s}$

The "no-load speed", ω_0, satisfies $\boxed{\frac{R_f v_a}{k' v_f} = \omega_0}$

$$\Rightarrow \frac{R_a R_f^2}{k k' v_f^2} T_s = \omega_0 = \overbrace{\frac{R_a R_f^2}{k k' v_f^2} T_m}^{\text{Divide by } \frac{R_a R_f^2}{k k' v_f^2} T_s = \omega_0} + \omega_m$$

$$\boxed{\Rightarrow 1 = \frac{T_m}{T_s} + \frac{\omega_m}{\omega_0}}$$

From previous slide:

$$1 = \frac{T_m}{T_s} + \frac{\omega_m}{\omega_0}$$

$$\Leftrightarrow \quad \omega_0 = \omega_0 \frac{T_m}{T_s} + \omega_m$$

$$\Leftrightarrow \quad \left(1 - \frac{T_m}{T_s}\right)\omega_0 = \omega_m$$

$$\Leftrightarrow \quad \omega_m = -\frac{\omega_0}{T_s} T_m + \omega_0 \qquad \Rightarrow$$

and:

$$\frac{R_f v_a}{k' v_f} = \omega_0$$

$$T_s = \frac{k\, v_a\, v_f}{R_f R_a}$$

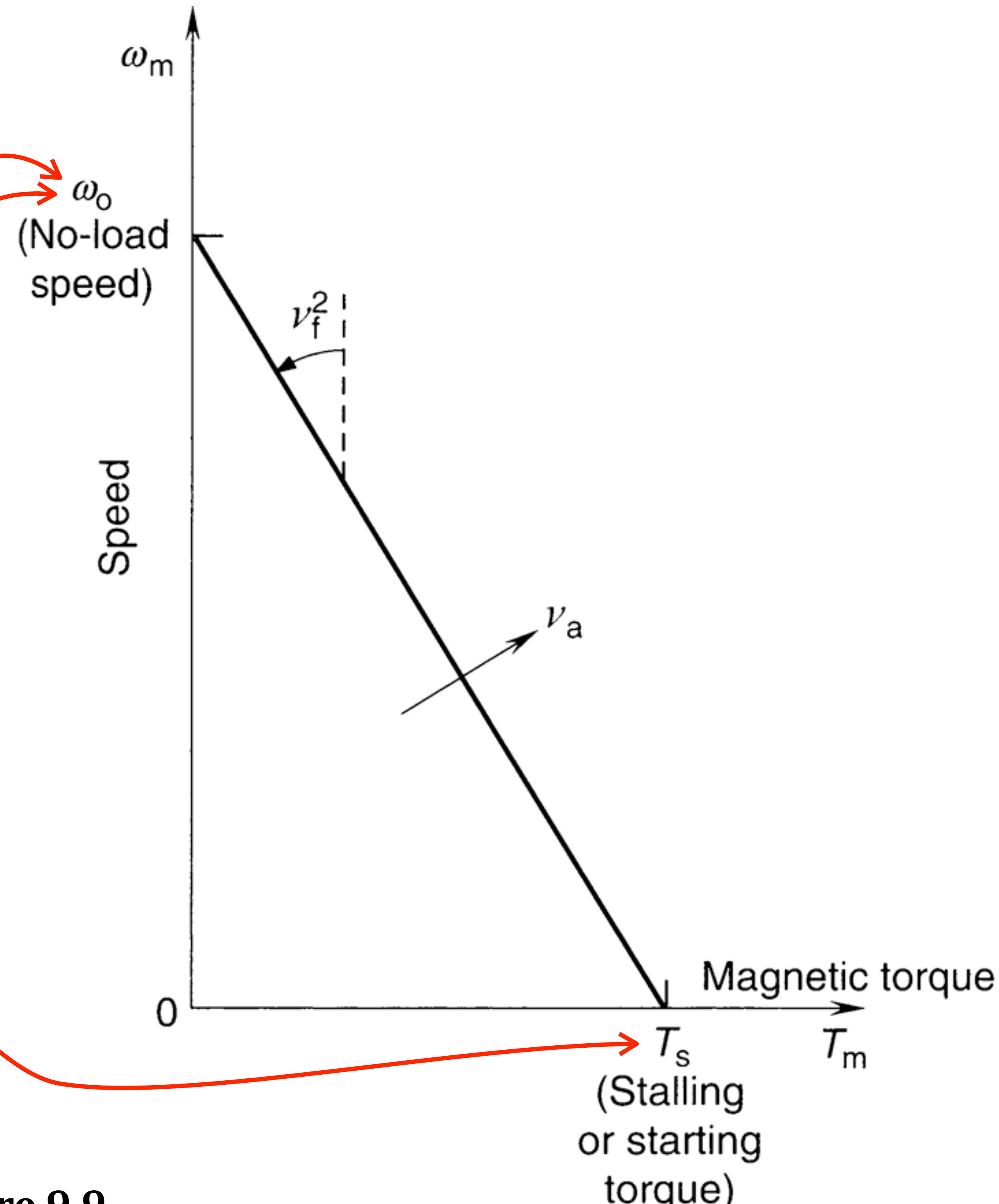


Figure 9.9
Steady-state speed–torque characteristics of a separately wound dc motor.