From previous slide:

$$1 = \frac{T_m}{T_s} + \frac{\omega_m}{\omega_0}$$

$$\Leftrightarrow \quad \omega_0 = \omega_0 \frac{T_m}{T_s} + \omega_m$$

$$\Leftrightarrow \quad \left(1 - \frac{T_m}{T_s}\right)\omega_0 = \omega_m$$

$$\Leftrightarrow \quad \omega_m = -\frac{\omega_0}{T_s} T_m + \omega_0 \quad \Rightarrow$$

and:

$$\frac{R_f v_a}{k' v_f} = \omega_0$$

$$T_s = \frac{k\, v_a\, v_f}{R_f R_a}$$

$$\text{Slope} = -\frac{\omega_0}{T_s} = -\frac{R_a R_f^2}{k k' v_f^2}$$

(Always negative with magnitude inversely proportional to $v_f^2$)

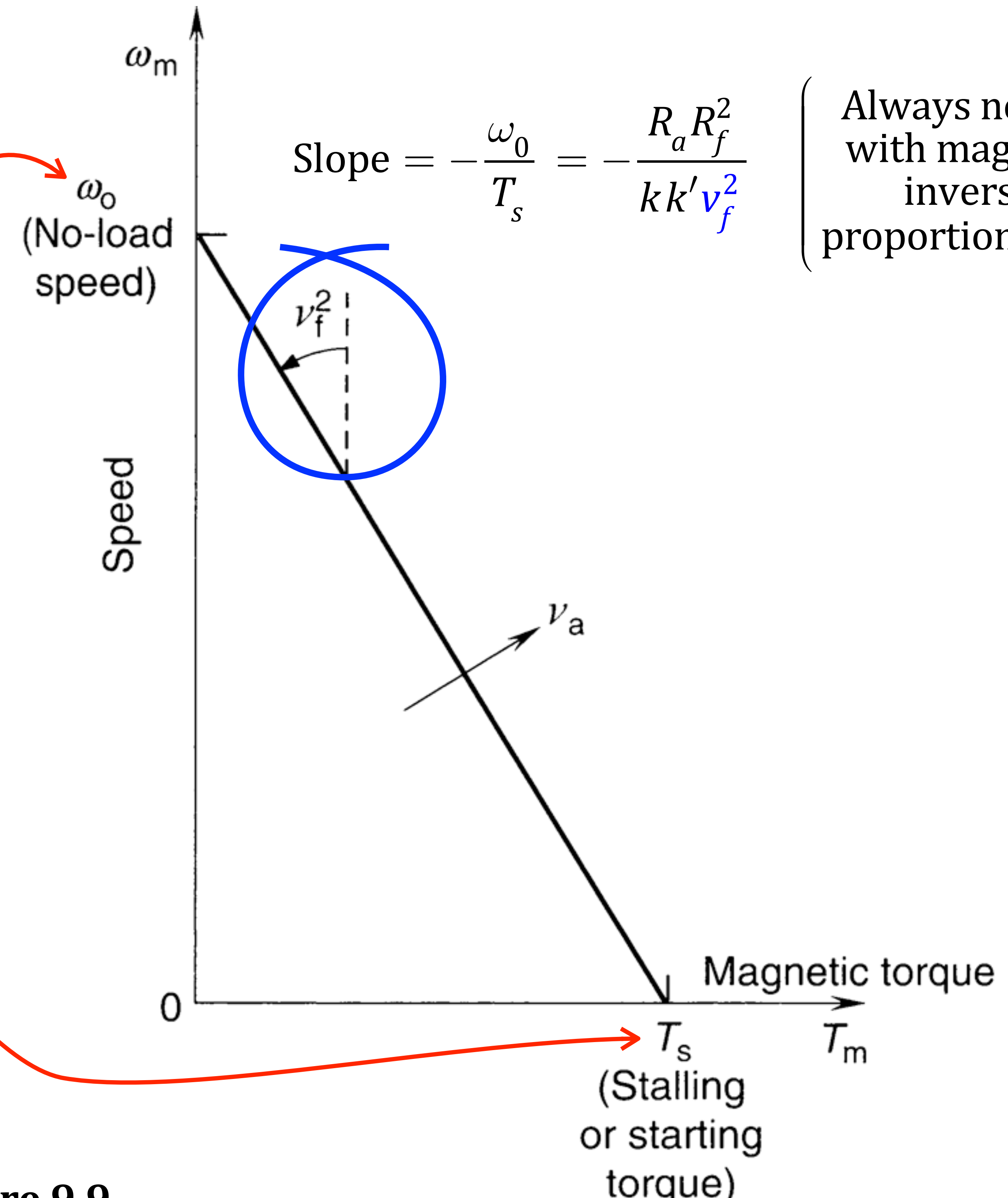



**Figure 9.9**
Steady-state speed–torque characteristics of a separately wound dc motor.

From previous slide:

$$1 = \frac{T_m}{T_s} + \frac{\omega_m}{\omega_0}$$

$$\Leftrightarrow \quad \omega_0 = \omega_0 \frac{T_m}{T_s} + \omega_m$$

$$\Leftrightarrow \quad \left(1 - \frac{T_m}{T_s}\right)\omega_0 = \omega_m$$

$$\Leftrightarrow \quad \omega_m = -\frac{\omega_0}{T_s} T_m + \omega_0 \quad \Rightarrow$$

and:

$$\frac{R_f v_a}{k' v_f} = \omega_0$$

$$T_s = \frac{k\, v_a v_f}{R_f R_a}$$

$$\text{Slope} = -\frac{\omega_0}{T_s} = -\frac{R_a R_f^2}{k k' v_f^2}$$

(Always negative with magnitude inversely proportional to $v_f^2$)

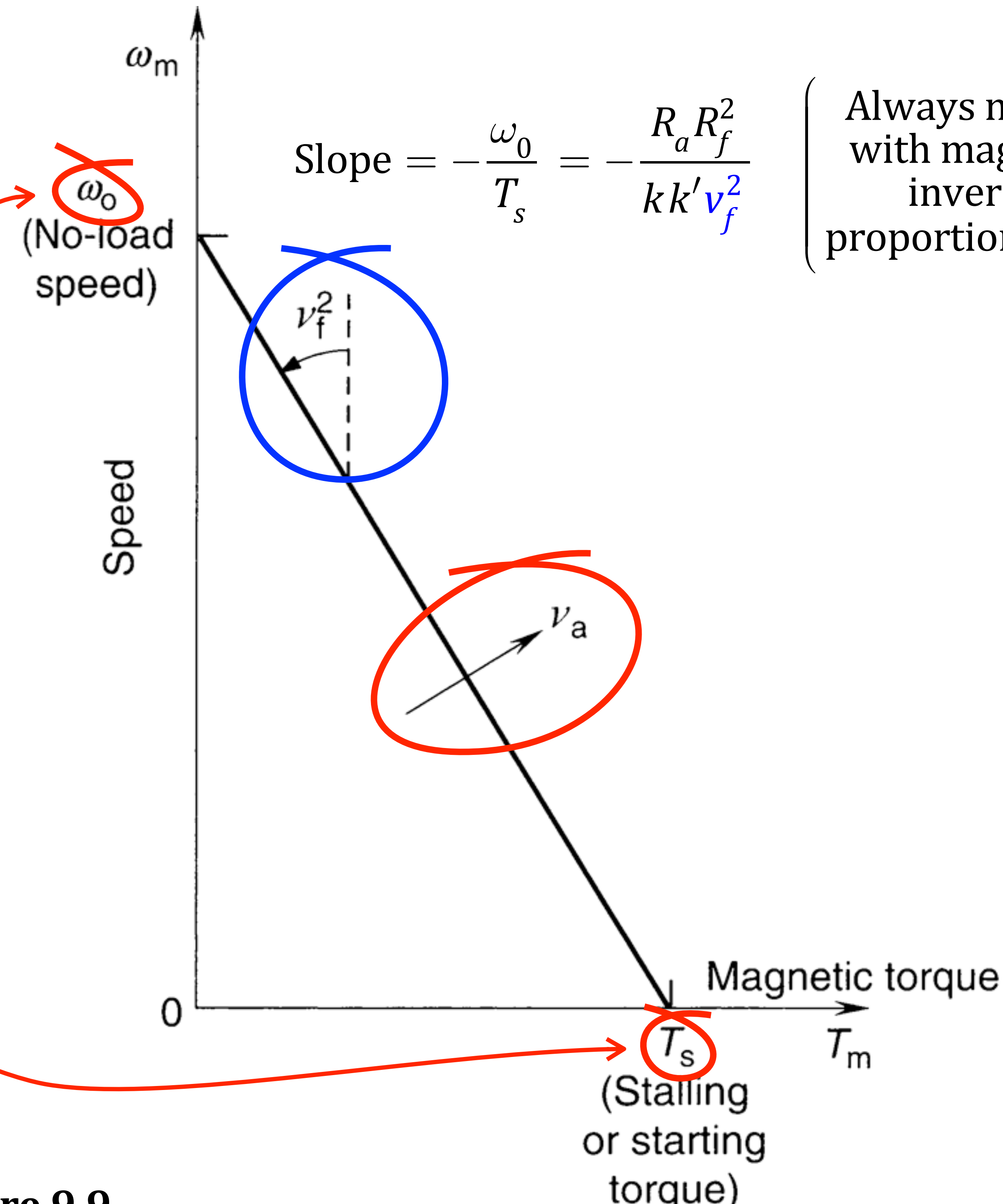



**Figure 9.9**
Steady-state speed–torque characteristics of a separately wound dc motor.

# Intuition for the motor torque-velocity curve

From previous slide:

$$1 = \frac{T_m}{T_s} + \frac{\omega_m}{\omega_0}$$

$$\Leftrightarrow \quad \omega_0 = \omega_0 \frac{T_m}{T_s} + \omega_m$$

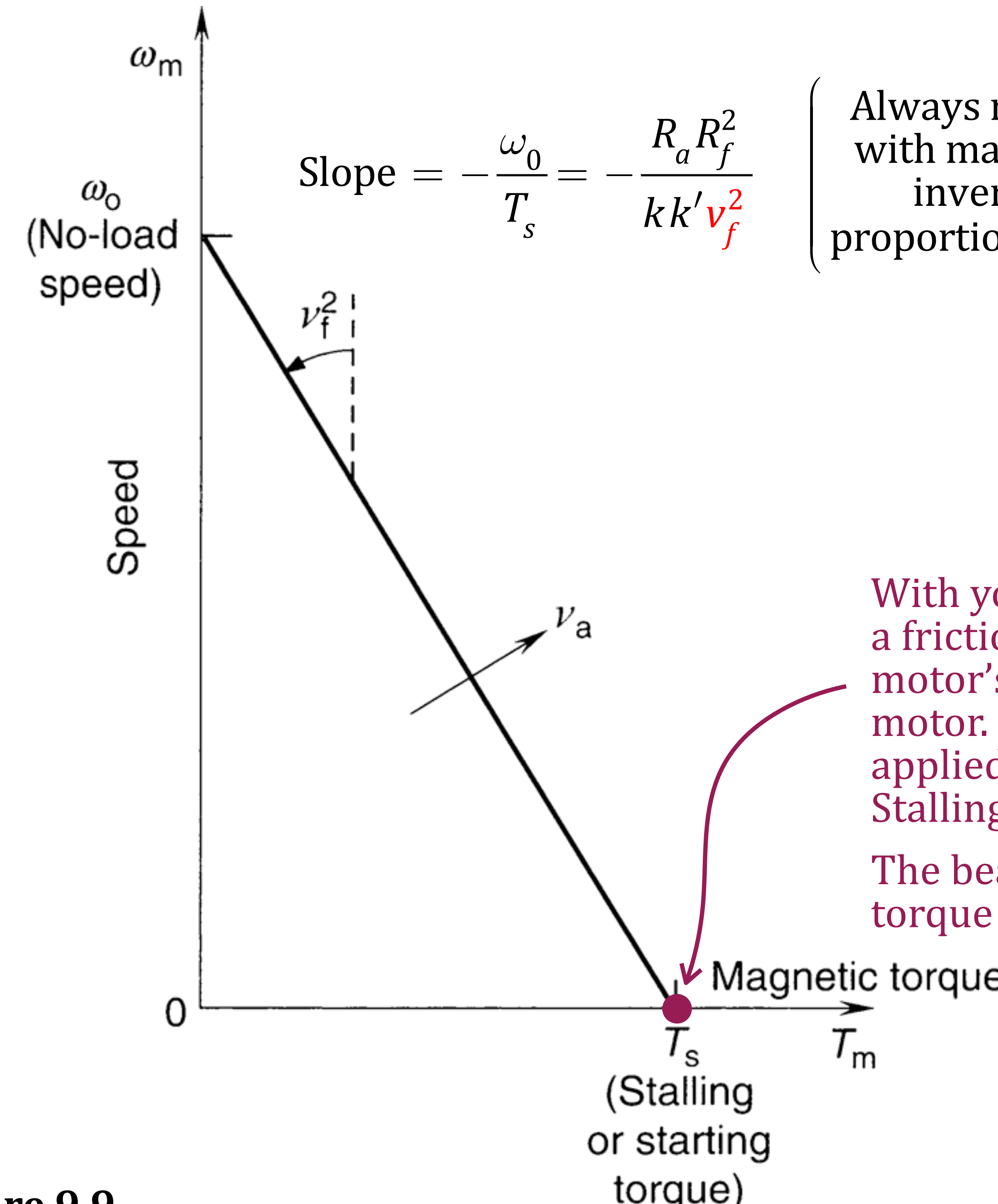
$$\Leftrightarrow \quad \left(1 - \frac{T_m}{T_s}\right)\omega_0 = \omega_m$$

$$\Leftrightarrow \quad \omega_m = -\frac{\omega_0}{T_s} T_m + \omega_0 \qquad \Rightarrow$$

and:

$$\frac{R_f v_a}{k' v_f} = \omega_0$$

$$T_s = \frac{k\, v_a v_f}{R_f R_a}$$




$$\text{Slope} = -\frac{\omega_0}{T_s} = -\frac{R_a R_f^2}{k k' v_f^2}$$

(Always negative with magnitude inversely proportional to $v_f^2$)

With your hands, apply a friction torque at the motor's shaft to stop the motor. The torque you've applied is the motor's Stalling Torque.

The bearing friction torque is now zero.

**Figure 9.9**
Steady-state speed–torque characteristics of a separately wound dc motor.

# Intuition for the motor torque-velocity curve

From previous slide:

$$1 = \frac{T_m}{T_s} + \frac{\omega_m}{\omega_0}$$

$$\Leftrightarrow \quad \omega_0 = \omega_0 \frac{T_m}{T_s} + \omega_m$$

$$\Leftrightarrow \quad \left(1 - \frac{T_m}{T_s}\right)\omega_0 = \omega_m$$

$$\Leftrightarrow \quad \omega_m = -\frac{\omega_0}{T_s} T_m + \omega_0 \quad \Rightarrow$$

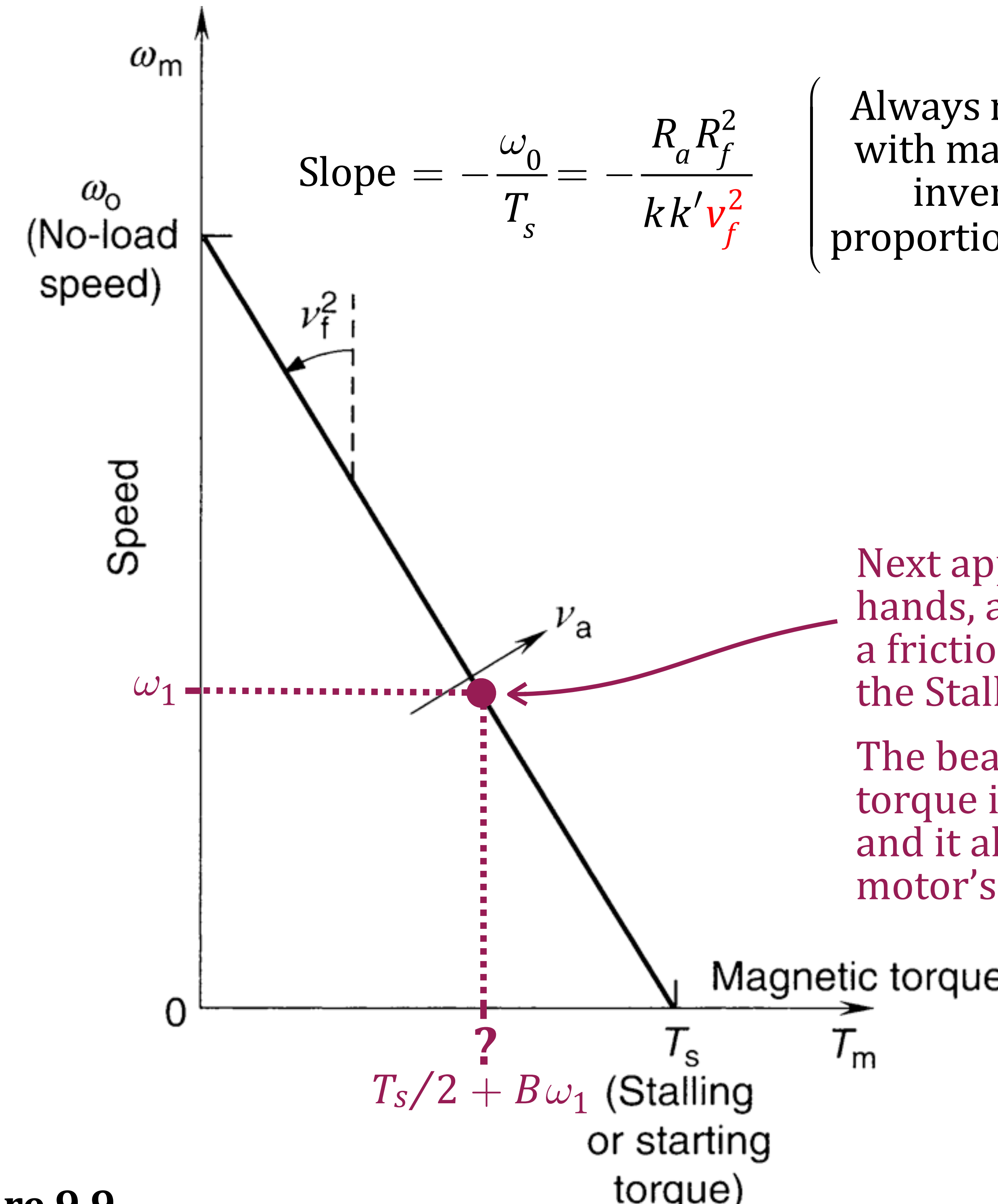
and:

$$\frac{R_f v_a}{k' v_f} = \omega_0$$

$$T_s = \frac{k\, v_a v_f}{R_f R_a}$$

$$\text{Slope} = -\frac{\omega_0}{T_s} = -\frac{R_a R_f^2}{k k' v_f^2}$$

(Always negative with magnitude inversely proportional to $v_f^2$)




Next apply, with your hands, at the motor's shaft, a friction torque that's half the Stalling Torque.

The bearing friction torque is now $-B\omega_1$, and it also opposes the motor's angular velocity.

**Figure 9.9**
Steady-state speed–torque characteristics of a separately wound dc motor.

Peak power output ($T_m \omega_m$) is at $\omega_m = \omega_o/2$

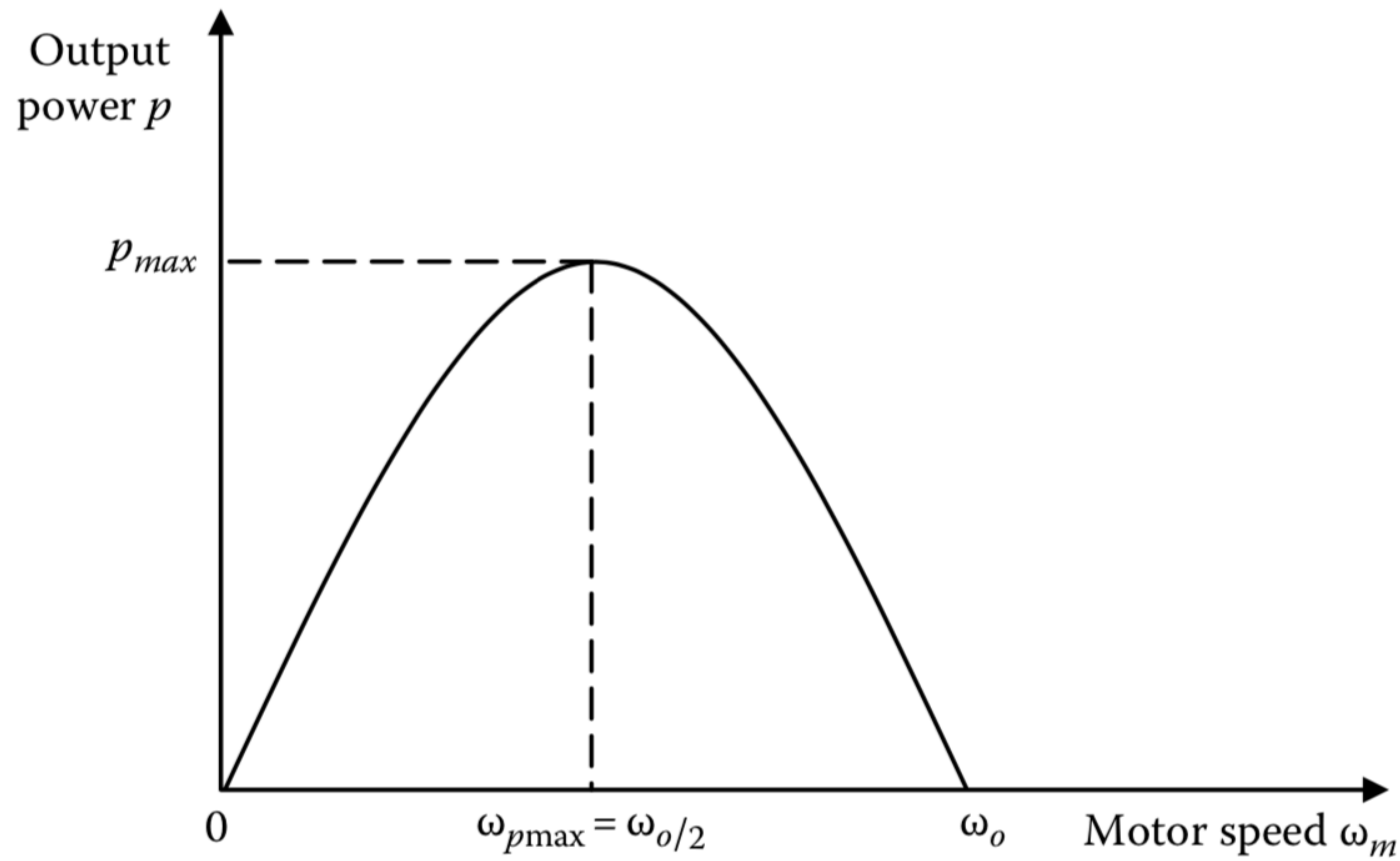



**FIGURE 9.11** Output power curve of a dc motor at steady state.

# Intuition for the motor torque-velocity curve

"No-load" here refers to a state where no torque whatsoever opposes the motor's rotation—not even a bearing friction torque.

$$1 = \frac{T_m}{T_s} + \frac{\omega_m}{\omega_0}$$

$$\Leftrightarrow \quad \omega_0 = \omega_0 \frac{T_m}{T_s} + \omega_m$$

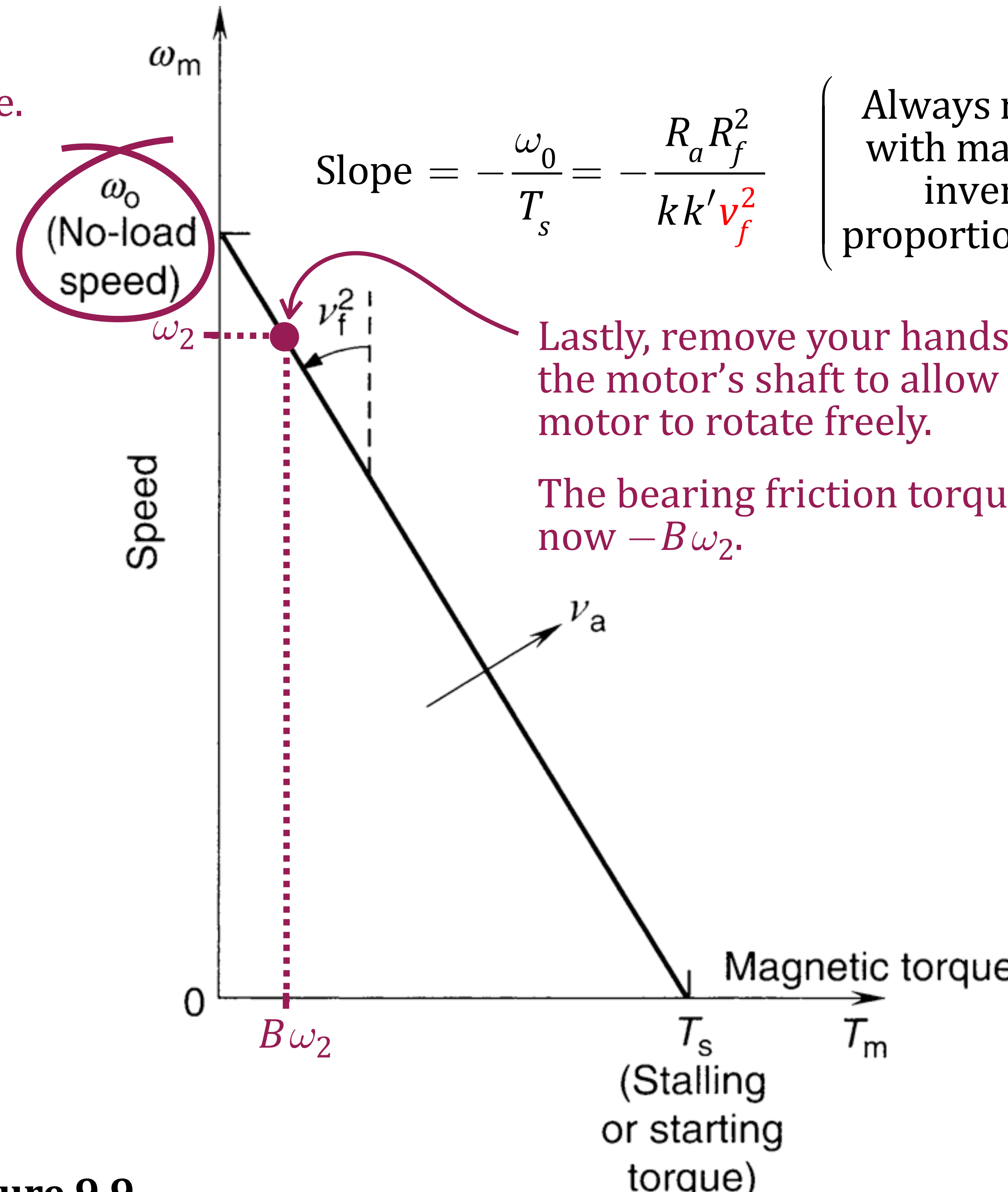
$$\Leftrightarrow \quad \left(1 - \frac{T_m}{T_s}\right)\omega_0 = \omega_m$$

$$\Leftrightarrow \quad \omega_m = -\frac{\omega_0}{T_s} T_m + \omega_0 \qquad \Rightarrow$$

and:

$$\frac{R_f v_a}{k' v_f} = \omega_0$$

$$T_s = \frac{k\, v_a\, v_f}{R_f R_a}$$




$$\text{Slope} = -\frac{\omega_0}{T_s} = -\frac{R_a R_f^2}{k k' v_f^2}$$

(Always negative with magnitude inversely proportional to $v_f^2$)

Lastly, remove your hands from the motor's shaft to allow the motor to rotate freely.

The bearing friction torque is now $-B\omega_2$.

**Figure 9.9**
Steady-state speed–torque characteristics of a separately wound dc motor.

# Armature-Controlled DC Motor dynamics

Dynamical equations:

$$T_m = k\, i_f\, i_a \triangleq k_m i_a$$

$$v_b = k' i_f \omega_m \triangleq k'_m \omega_m$$

$$k = k'$$

$$v_f = R_f i_f + L_f \frac{di_f}{dt}$$

$$v_a = R_a i_a + L_a \frac{di_a}{dt} + v_b$$

$$J_m \frac{d\omega_m}{dt} = T_m - T_L - b_m \omega_m$$

$\Rightarrow$

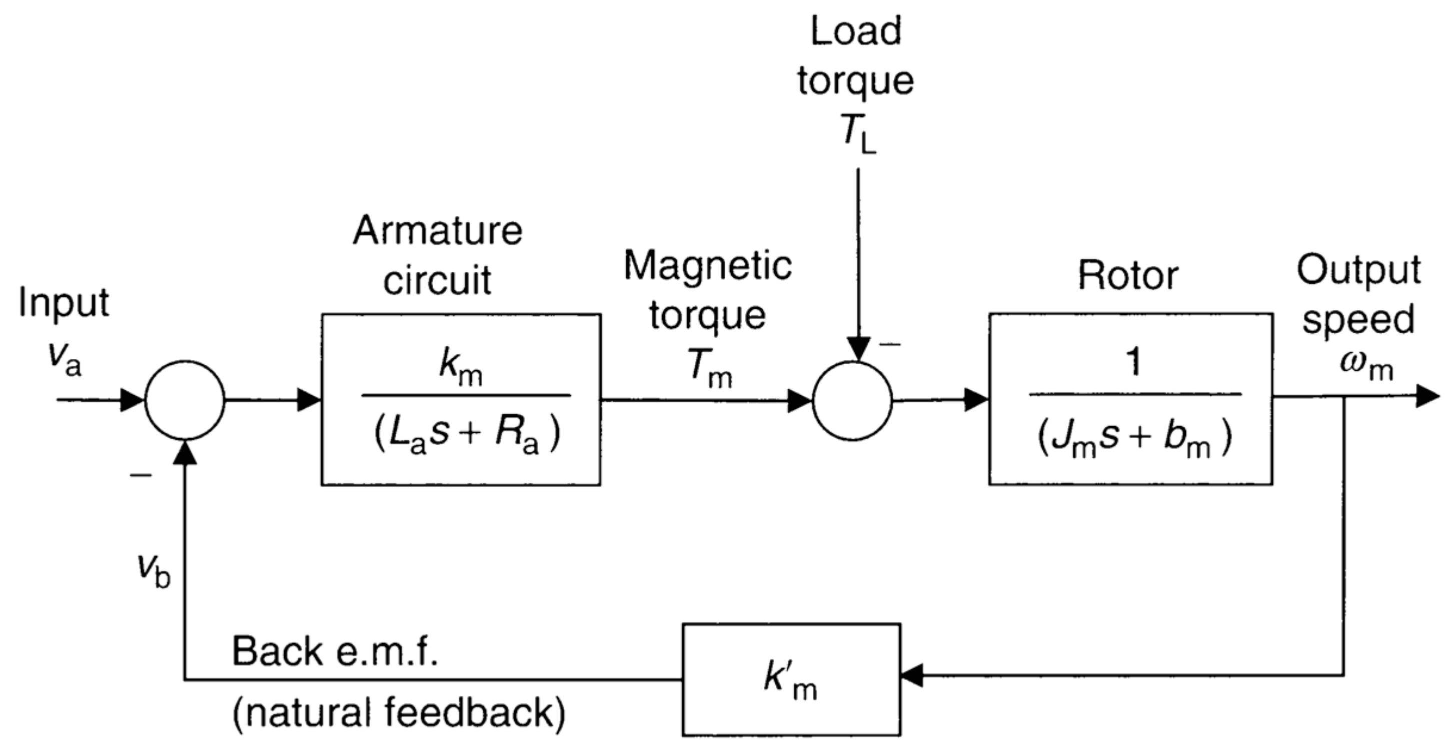
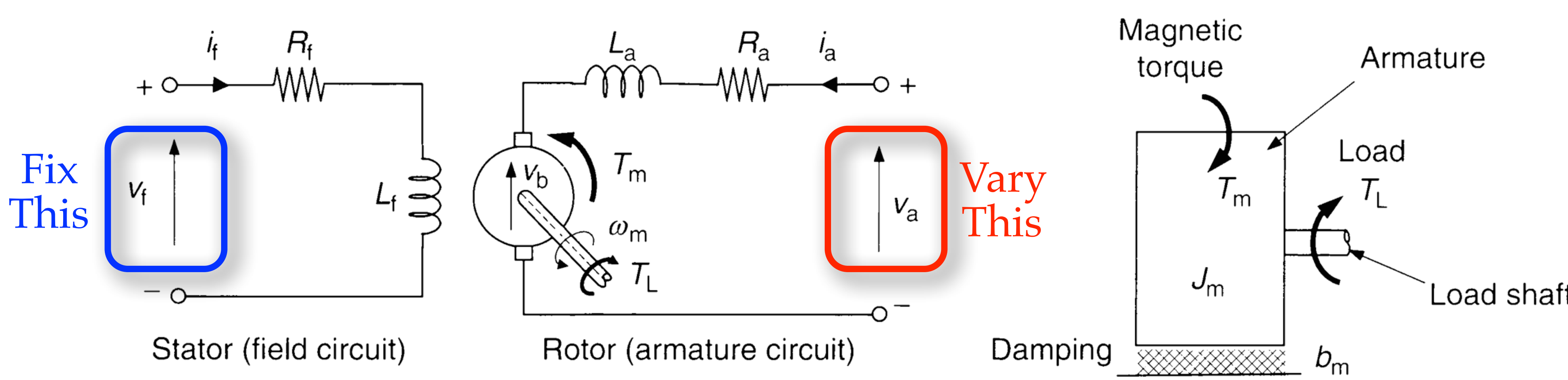



**Figure 9.16**
block-diagram representation of the dynamics of an armature-controlled DC motor in open loop



# Field-Controlled DC Motor

Dynamical Equations:

$$T_m = k i_f i_a \triangleq k_a i_f$$

$$v_b = k' i_f \omega_m$$

$$k = k'$$

$$v_f = R_f i_f + L_f \frac{di_f}{dt}$$

$$v_a = R_a i_a + L_a \frac{di_a}{dt} + v_b$$

$$J_m \frac{d\omega_m}{dt} = T_m - T_L - b_m \omega_m$$

$\Rightarrow$

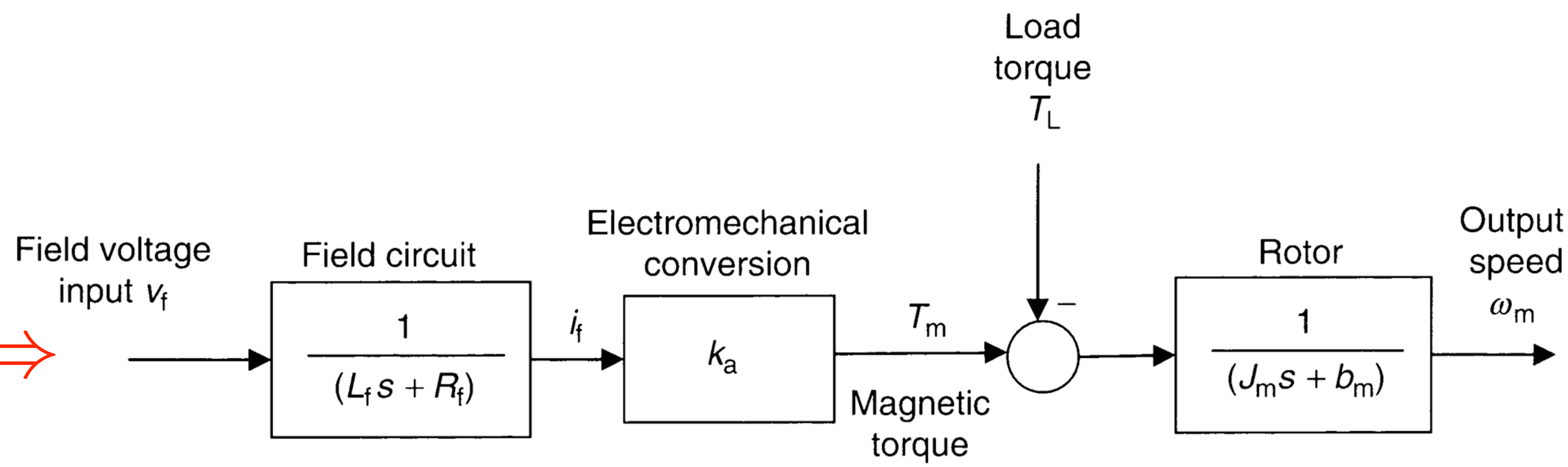
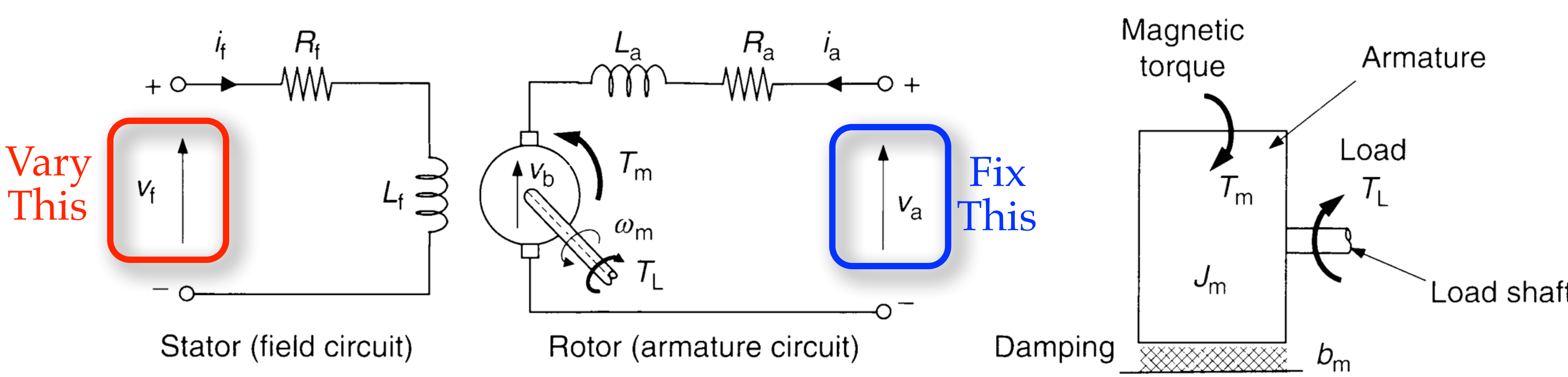
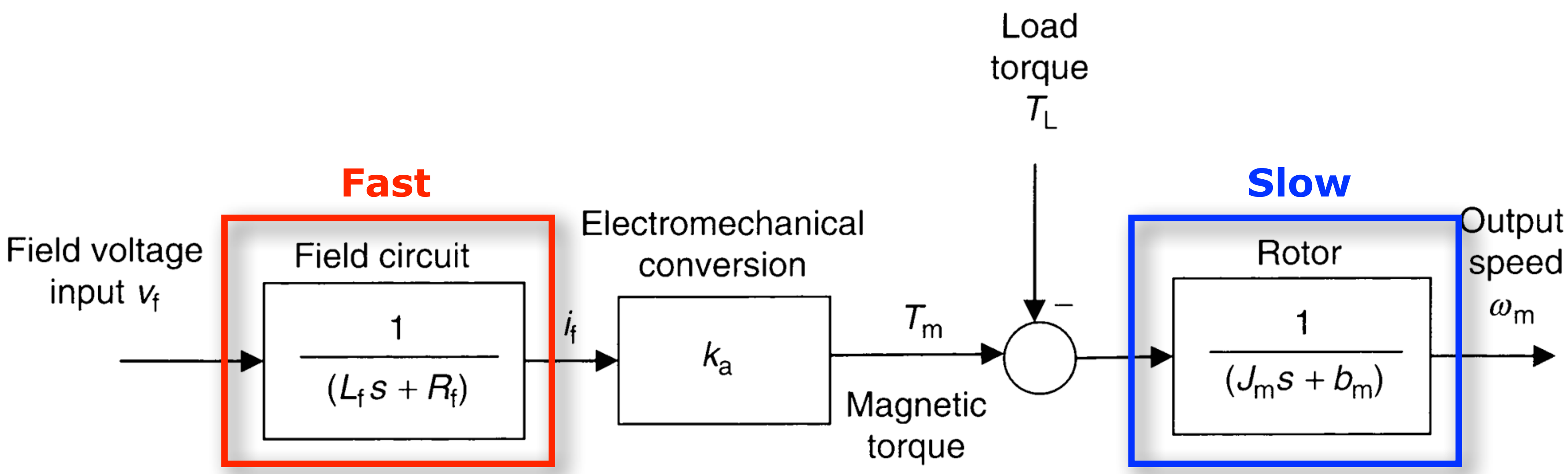



**Figure 9.21**
Open-loop block diagram for a field-controlled dc motor.



# Field-Controlled DC Motor




Dynamics:

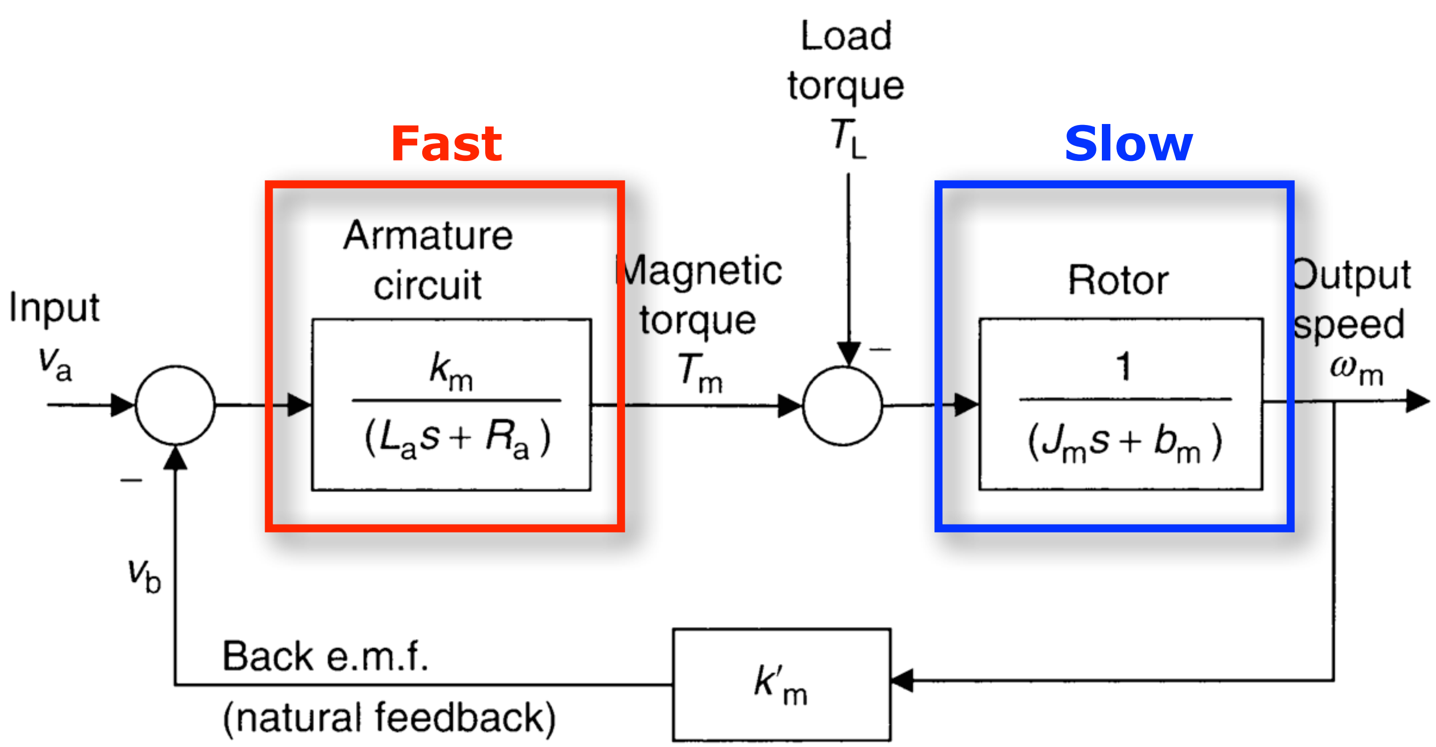
$$\Omega_m(s) = \frac{k_a}{(L_f s + R_f)(J_m s + b_m)} V_f(s) - \frac{1}{(J_m s + b_m)} T_L(s)$$

$$= \frac{\frac{k_a}{R_f b_m}}{\left(\frac{L_f}{R_f}s+1\right)\left(\frac{J_m}{b_m}s+1\right)} V_f(s) - \frac{\frac{1}{b_m}}{\left(\frac{J_m}{b_m}s+1\right)} T_L(s)$$

Typically $\frac{J_m}{b_m} \gg \frac{L_f}{R_f}$, so that

$$\Omega_m(s) \approx \frac{\frac{k_a}{R_f b_m}}{\left(\frac{J_m}{b_m}s+1\right)} V_f(s) - \frac{\frac{1}{b_m}}{\left(\frac{J_m}{b_m}s+1\right)} T_L(s)$$

# Armature-Controlled DC Motor




Dynamics:

$$\Omega_m(s) = \frac{k_m}{\Delta(s)} V_a(s) - \frac{L_a s + R_a}{\Delta(s)} T_L(s)$$

$$= \frac{k_m}{\Delta(s)} V_a(s) - \frac{R_a\left(\frac{L_a}{R_a} s + 1\right)}{\Delta(s)} T_L(s)$$

$$\Delta(s) = (L_a s + R_a)(J_m s + b_m) + k_m k'_m$$

$$= R_a b_m \left(\frac{L_a}{R_a} s + 1\right)\left(\frac{J_m}{b_m} s + 1\right) + k_m k'_m$$

Typically $\frac{J_m}{b_m} \gg \frac{L_a}{R_a}$, and then**

$$\Omega_m(s) \approx \frac{k_m}{\tilde{\Delta}(s)} V_a(s) - \frac{R_a}{\tilde{\Delta}(s)} T_L(s)$$

$$\tilde{\Delta}(s) = R_a b_m \left(\frac{J_m}{b_m} s + 1\right) + k_m k'_m$$

** To see this, think about the Bode plot of $\left(\frac{L_a}{R_a} s + 1\right)\left(\frac{J_m}{b_m} s + 1\right)$.

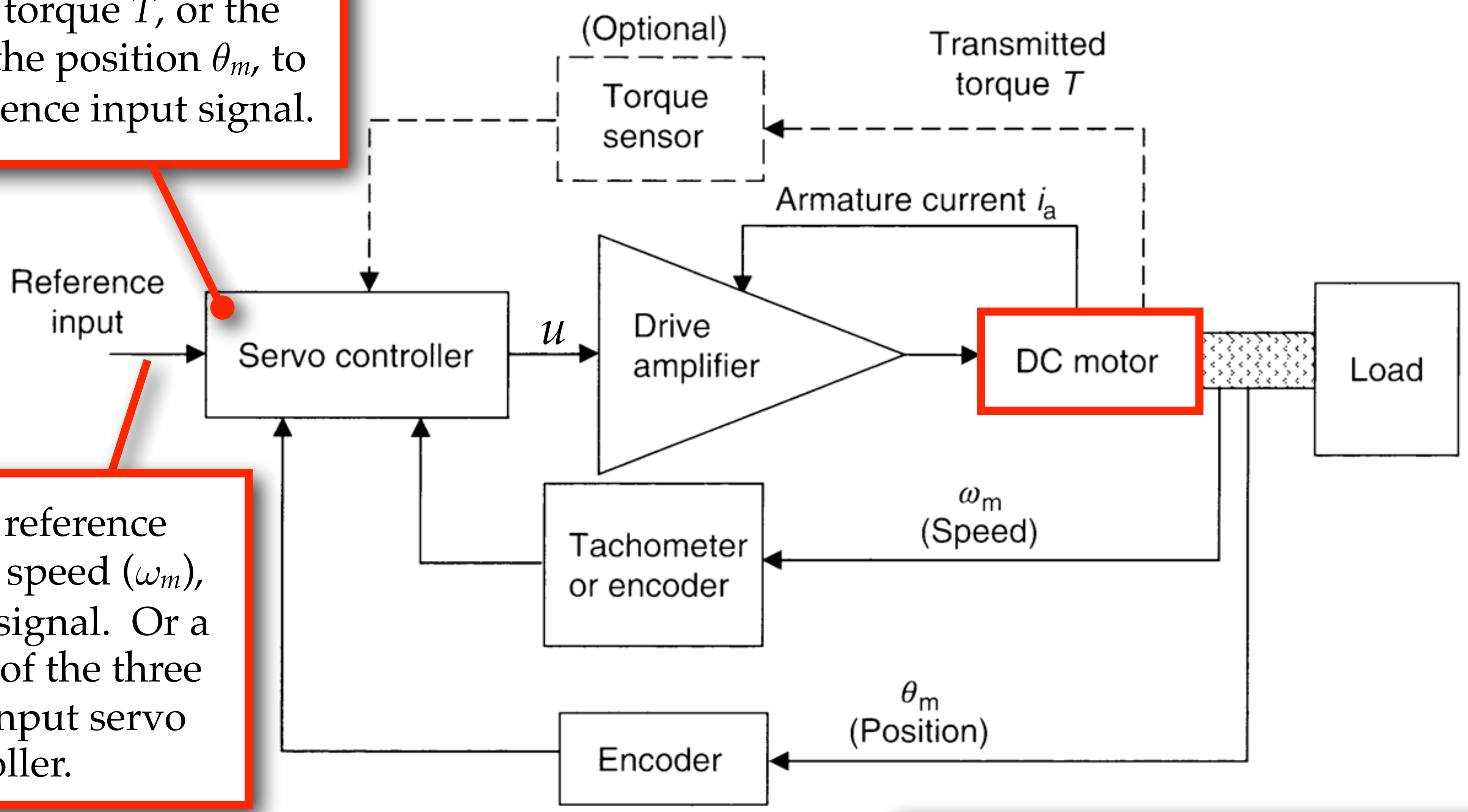
# DC Motor Selection

Motor manufacturers' data that are usually available to users include the following:

1. Mechanical data
   (a) Peak torque (e.g., 65 N.m)
   (b) Continuous torque at zero speed or continuous stall torque (e.g., 25 N.m)
   (c) Frictional torque (e.g., 0.4 N.m)
   (d) Maximum acceleration at peak torque (e.g., $33 \times 10^3$ rad/s$^2$)
   (e) Maximum speed or no-load speed (e.g., 3000 rpm)
   (f) Rated speed or speed at rated load (e.g., 2400 rpm)
   (g) Rated output power (e.g., 5100 W)
   (h) Rotor moment of inertia (e.g., 0.002 kg.m$^2$)
   (i) Dimensions and weight (e.g., 14 cm diameter, 30 cm length, 20 kg)
   (j) Allowable axial load or thrust (e.g., 230 N)
   (k) Allowable radial load (e.g., 700 N)
   (l) Mechanical (viscous) damping constant (e.g., 0.12 N.m/krpm)
   (m) Mechanical time constant (e.g., 10 m.s)
2. Electrical data
   (a) Electrical time constant (e.g., 2 m.s)
   (b) Torque constant (e.g., 0.9 N.m/A for peak current or 1.2 N.m/A rms current)
   (c) Back e.m.f. constant (e.g., 0.95 V/rad/s for peak voltage)
   (d) Armature/field resistance and inductance (e.g., 1.0 Ω, 2 mH)
   (e) Compatible drive unit data (voltage, current, etc.)
3. General data
   (a) Brush life and motor life (e.g., $5 \times 10^8$ revolutions at maximum speed)
   (b) Operating temperature and other environmental conditions (e.g., 0 to 40°C)
   (c) Thermal resistance (e.g., 1.5°C/W)
   (d) Thermal time constant (e.g., 70 min)
   (e) Mounting configuration

# DC Servomotor

Typically, filtering circuitry to condition the sensor input signals + PID controller circuitry to determine the output signal $u$ to cause the transmitted torque $T$, or the speed $\omega_m$, or the position $\theta_m$, to track the reference input signal.



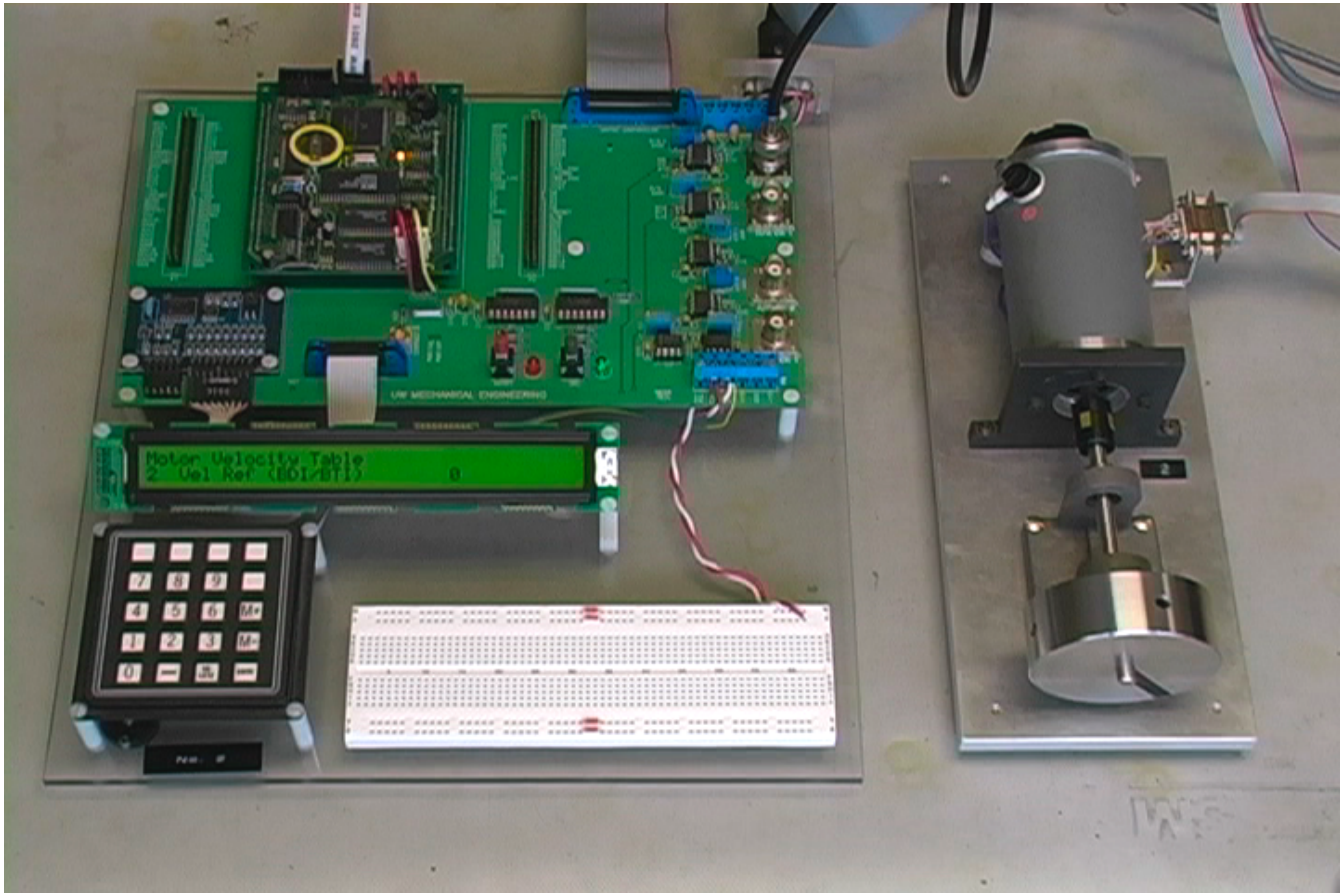

Typically a reference position ($\theta_m$), speed ($\omega_m$), or torque ($T$) signal. Or a combination of the three for a multi-input servo controller.

**Figure 9.15**
A dc servomotor system.

A DC servomotor is a DC motor with feedback control "built in" to make the motor follow a specified motion or torque trajectory.

# A DC Servomotor for Angular Velocity

## Closed-Loop Control of DC Motor Angular Velocity

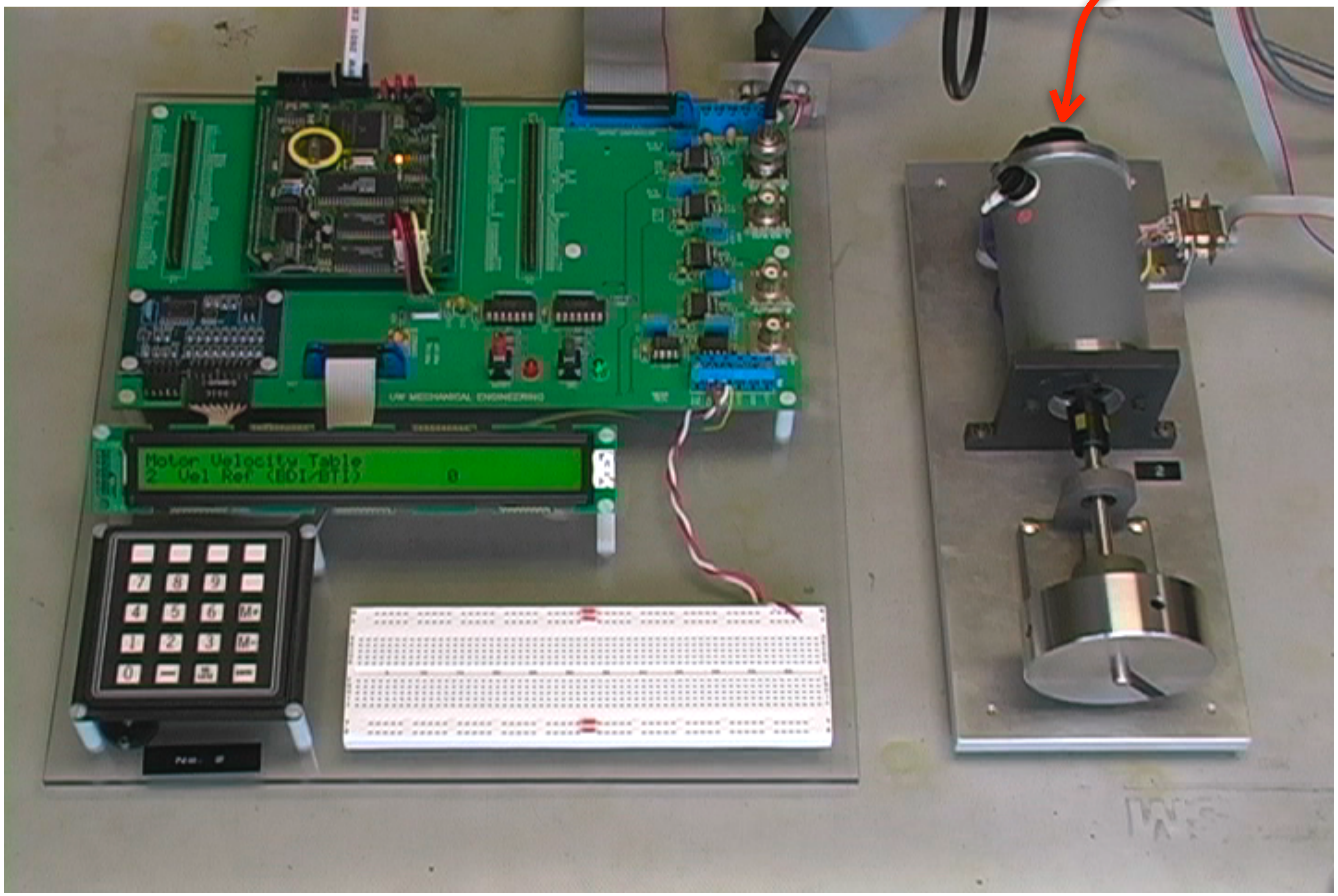



Start/End: Space Bar  Pause/Resume: K  Rewind: J  Fast-Forward: L  Jump to Beginning: I  Jump to End: O

# A DC Servomotor for Angular Velocity

## Closed-Loop Control of DC Motor Angular Velocity

Optical Encoder




Counter Chip for Optical Encoder

Power Supply for DC Motor