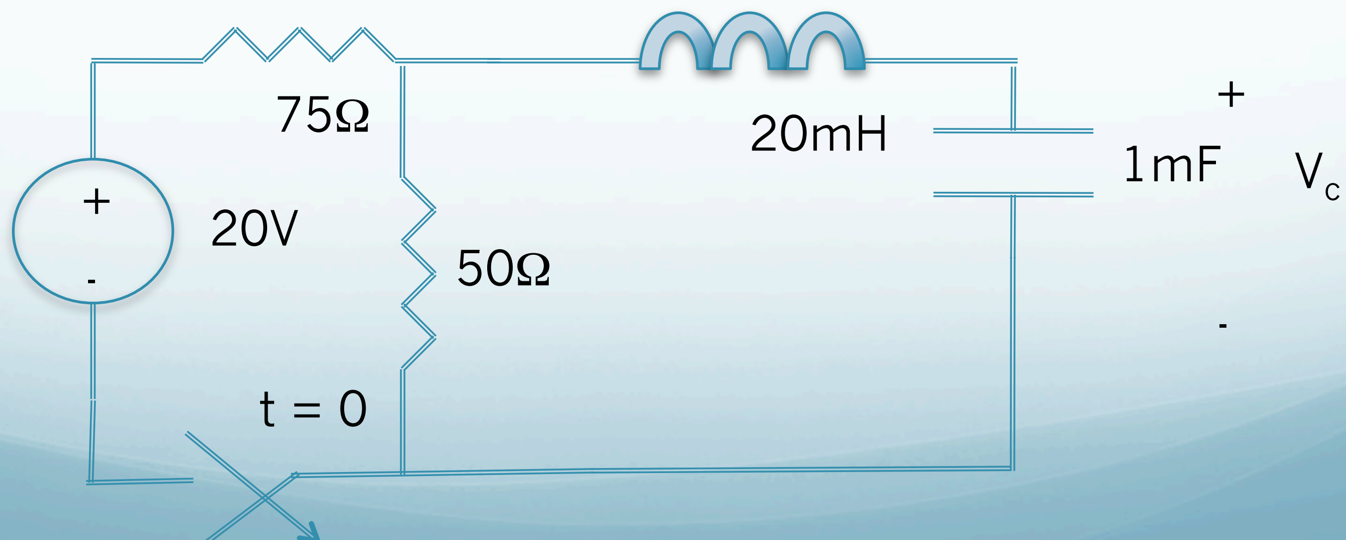
Introduction to Electronic Circuits

DC Circuit Analysis: Transient Response of RLC Circuits

DC Circuits: Transient Analysis

Up until this point, we have been looking at the "Steady State" response of DC circuits. "Steady-State" implies that nothing has changed in the circuit in a long time – no power has been turned on or off, no switches have been flipped, no lightening bolts have hit the circuit, and so on.

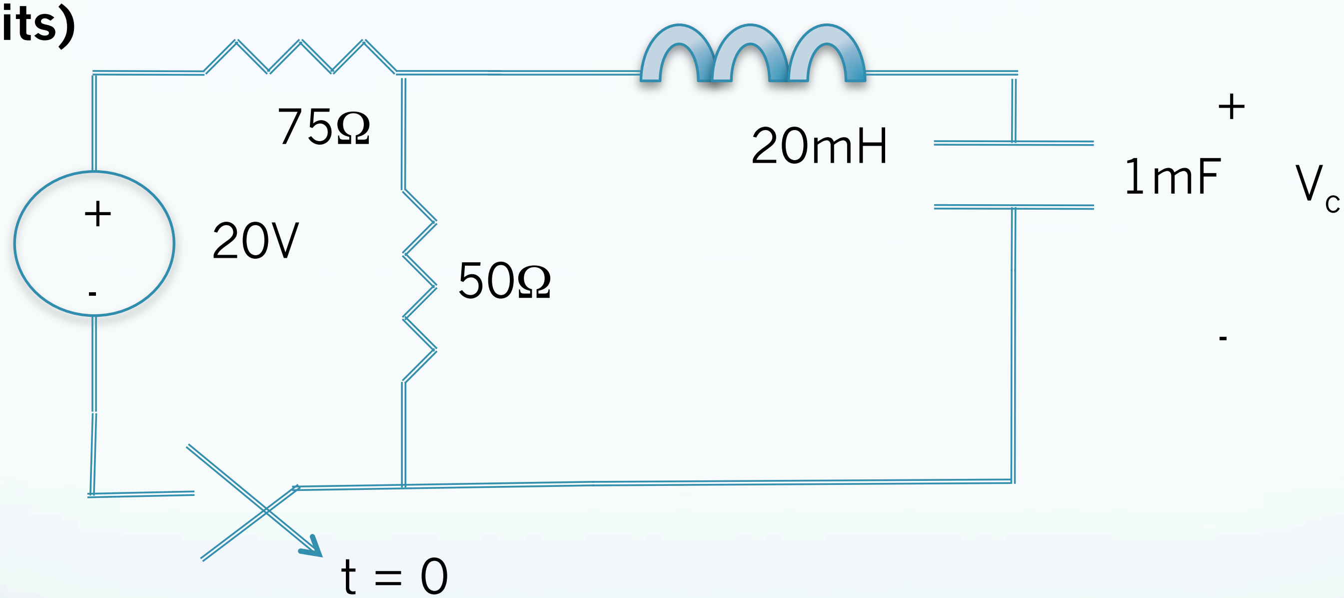
In DC "Steady State" conditions, we can treat inductors as short circuits and capacitors as open circuits. However, when a switch has just been "flipped" from on to off or off to on, we can no longer make these assumptions about inductors and capacitors. The time-dependent or DC "Transient" response of circuits must then be calculated using known behaviors of inductors (V = L di/dt) and capacitors (i = C dv/dt) and the appropriate mathematics/ calculus.



DC Circuits: Transient Analysis

Our discussion of transient response is divided into several sections:

1. Those circuits that contain only resistors and capacitors (RC circuits)
2. Those circuits that contain only resistors and inductors (RL circuits)
3. **Those circuits that contain resistors, capacitors, and inductors (RLC circuits)**



and

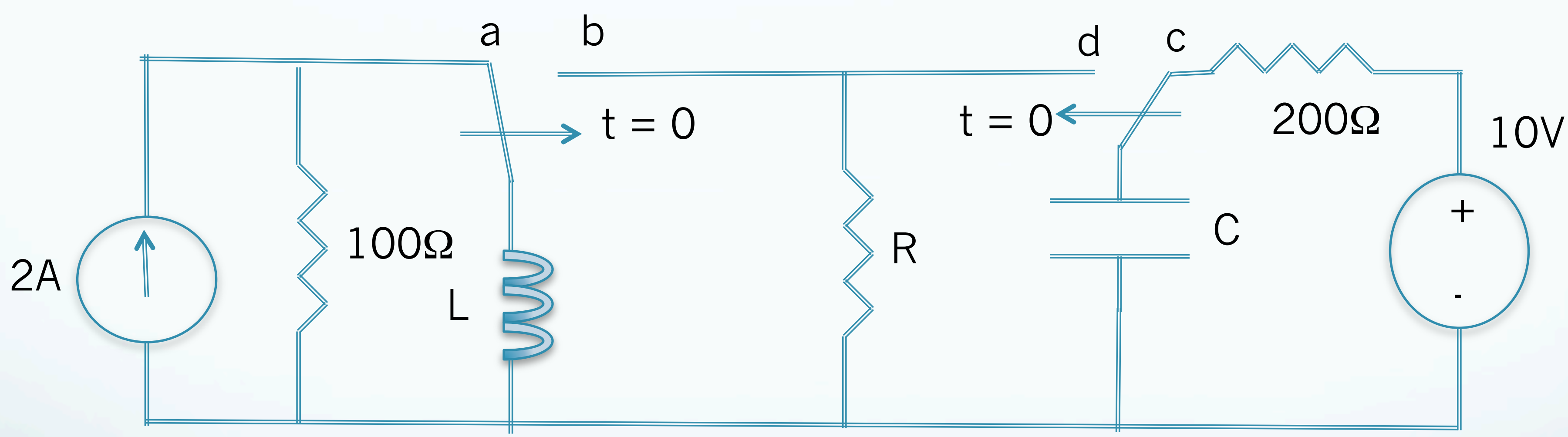
1. **Those circuits responding to power removed from the circuit (Natural Response)**
2. **Those circuits responding to power delivered/applied to the circuit (Step Response)**

The circuit above is an RLC circuit and a Natural Response

DC Circuits: Transient Analysis

In this lecture, we will look at the following circuits:

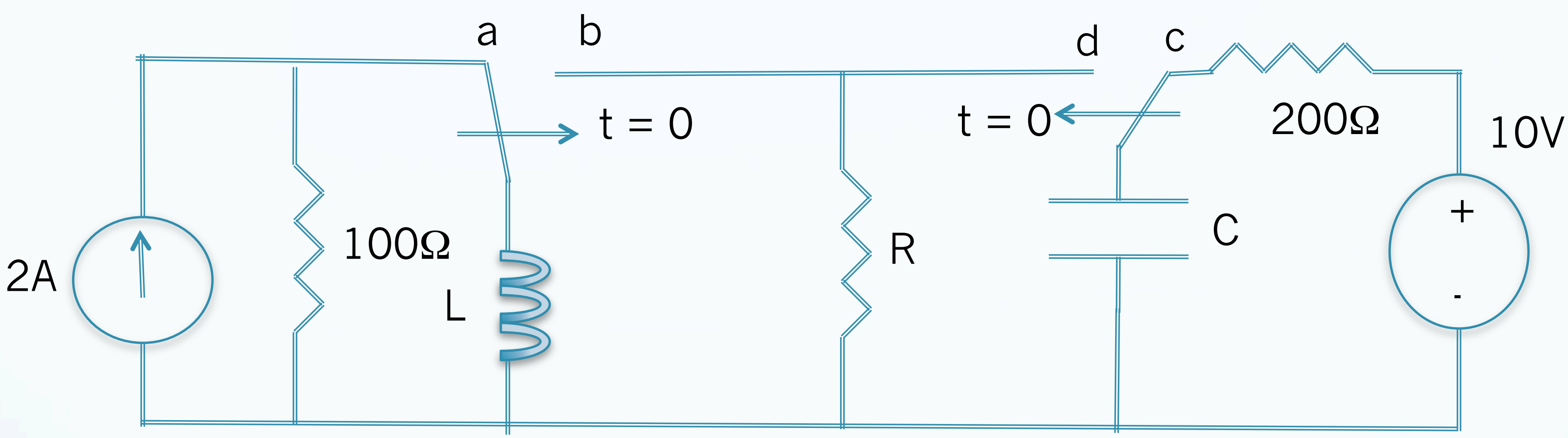
1. **Those circuits that contain resistors, inductors, and capacitors (RLC Circuits)**
2. **Those circuits responding to power removed from the circuit (Natural Response)**
3. Those circuits responding to power delivered/applied to the circuit (Step Response)



This circuit is an RLC circuit
which goes into its Natural Response at t > 0

DC Circuits: Transient Analysis

1. **Those circuits that contain resistors, inductors, and capacitors (RLC Circuits)**
2. **Those circuits responding to power removed from the circuit (Natural Response)**



Right before t = 0 (at t = 0-):
The circuit is presumed to have had the switch in position A and the switch in position C for "a long time" and is in DC Steady State; the inductor is acting like a short circuit and the capacitor like a short circuit.

	R	**L**	**C**
Voltage	0	0	10V
Current	0	2A	0

DC Circuits: Transient Analysis

1. **Those circuits that contain resistors, inductors, and capacitors (RLC Circuits)**
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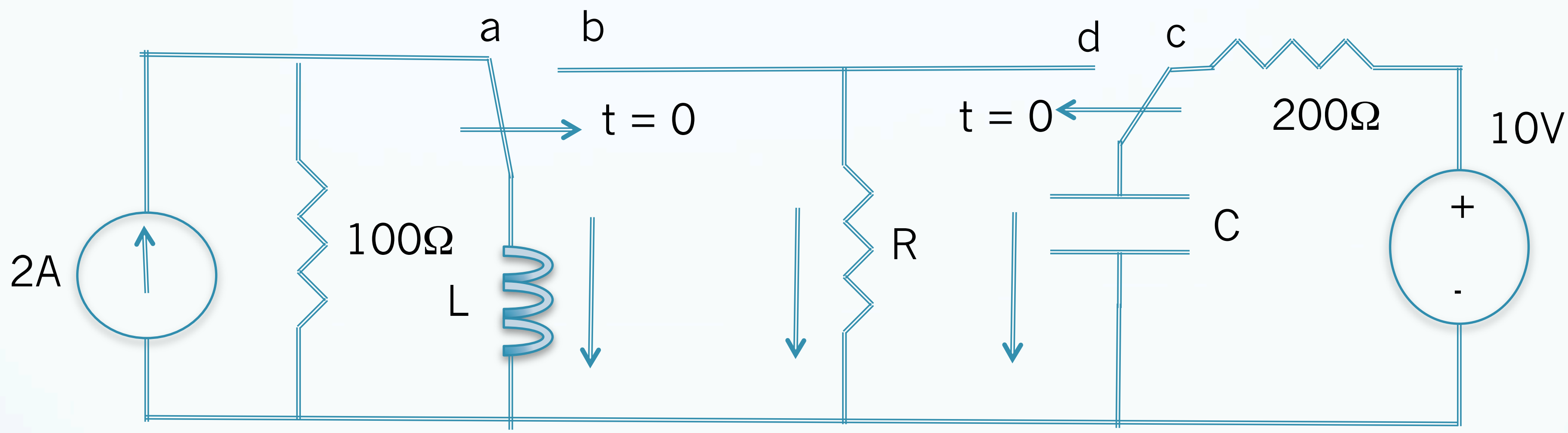
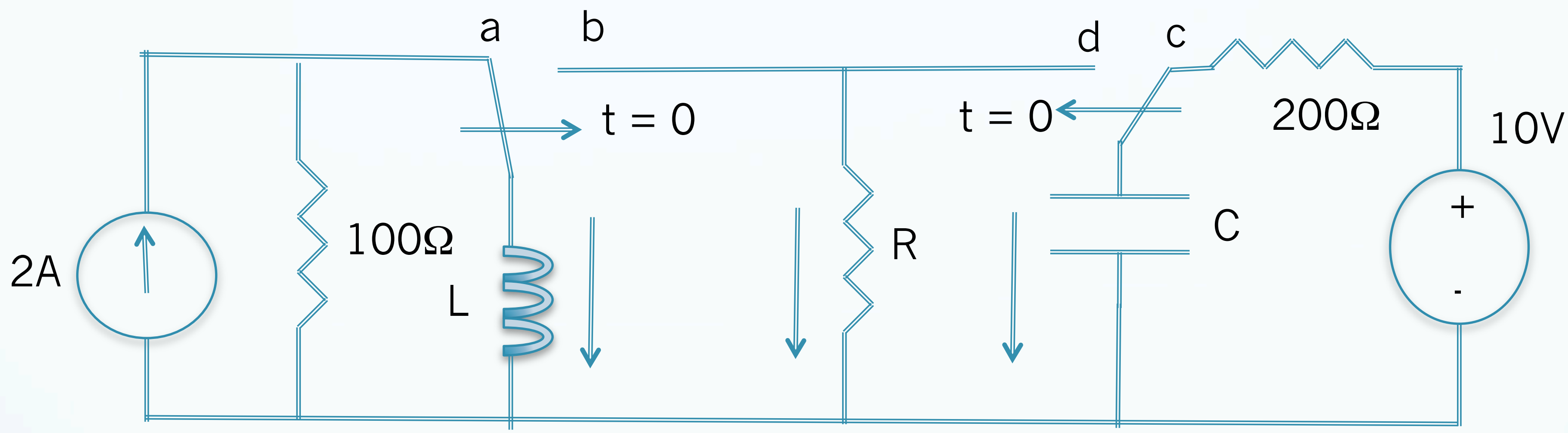


Right after t = 0 (at t = 0+):
Neither the capacitor voltage nor the inductor current can change instantaneously; we use this fact to calculate the current through and the voltage across the resistor R.

	R	**L**	**C**
Voltage	10	10	10
Current	10/R	2	-(10/R + 2)

DC Circuits: Transient Analysis

1. **Those circuits that contain resistors, inductors, and capacitors (RLC Circuits)**
2. **Those circuits responding to power removed from the circuit (Natural Response)**

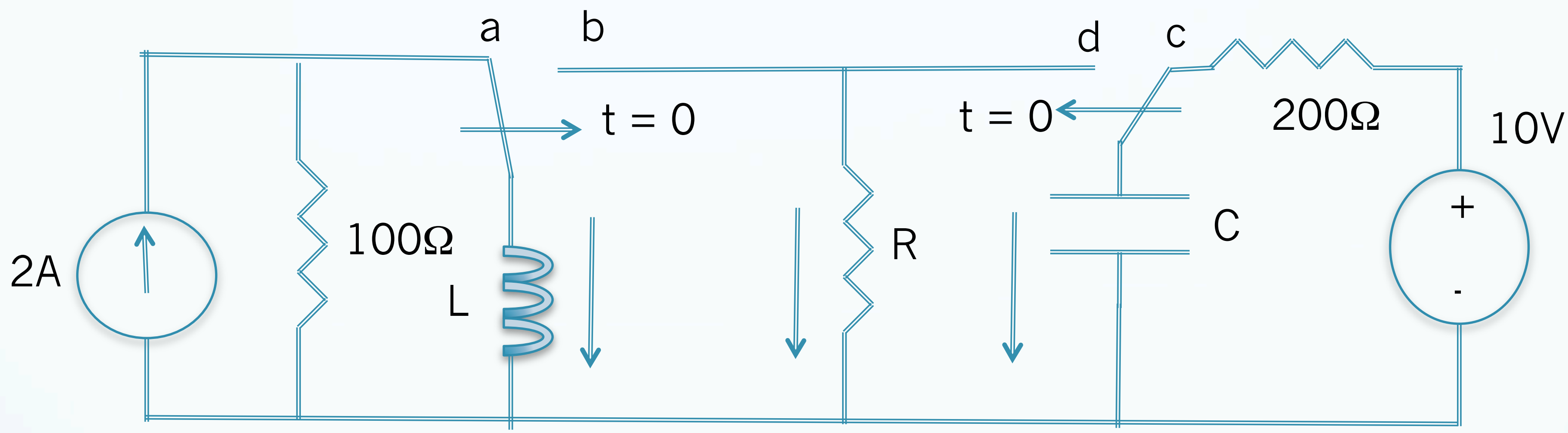


After the switch is in position b for a long time, and
After the switch in position d for a long time.
The capacitor becomes an open circuit.
The inductor becomes a short circuit.

	R	L	C
Voltage	0	0	0
Current	0	0	0

DC Circuits: Transient Analysis

1. **Those circuits that contain resistors, inductors, and capacitors (RLC Circuits)**
2. **Those circuits responding to power removed from the circuit (Natural Response)**



At $t > 0$, KCL gives:
$I_L + I_R + I_C + Io = 0$

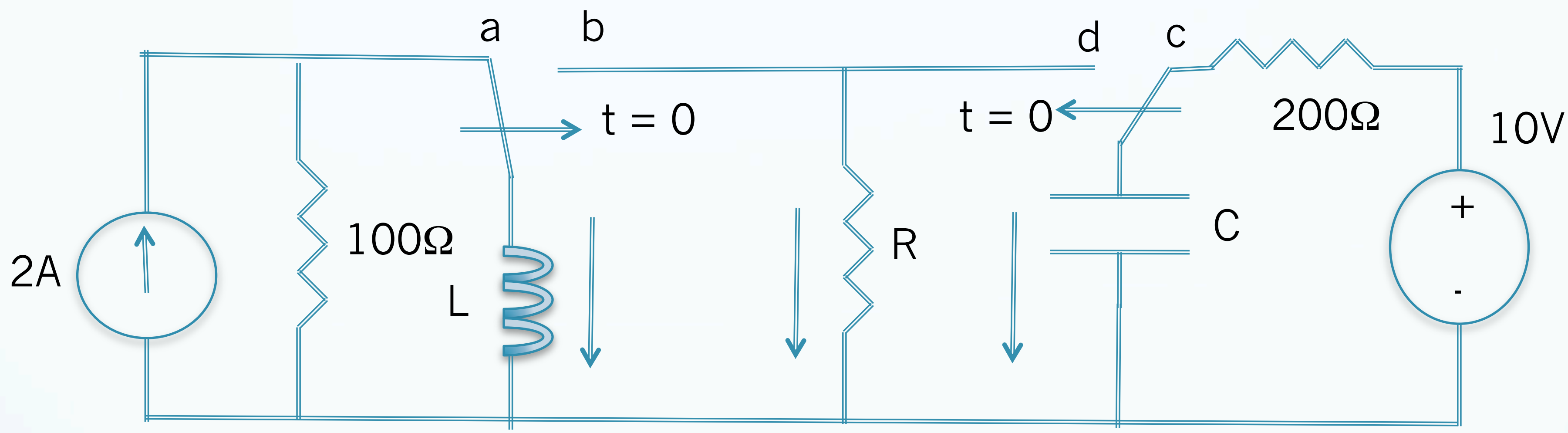
Substituting governing relationships for R, L, and C gives:

$$C\, dV/dt + V/R + 1/L \int V\, dt + Io = 0$$

where V is the voltage across the inductor, capacitor, and resistor
Io is the initial current flowing through the inductor (the resistor and capacitor have no current flowing through them at $t = 0_-$)

DC Circuits: Transient Analysis

1. **Those circuits that contain resistors, inductors, and capacitors (RLC Circuits)**
2. **Those circuits responding to power removed from the circuit (Natural Response)**



Taking the derivative of the following expression:

$$C\, dV/dt + V/R + 1/L \int V\, dt + I_o = 0$$

gives a second order differential equation:

$$(C)\, d^2V/dt^2 + (1/R)\, dV/dt + (1/L)\, V = 0$$

or:

$$d^2V/dt^2 + (1/RC)\, dV/dt + (1/LC)\, V = 0$$

DC Circuits: Transient Analysis

1. **Those circuits that contain resistors, inductors, and capacitors (RLC Circuits)**
2. **Those circuits responding to power removed from the circuit (Natural Response)**

Natural Response of the parallel RLC circuit:

Solving the second order differential equation:
$d^2V/dt^2 + (1/RC)\ dV/dt + (1/LC)\ V = 0$

Requires finding the roots of the characteristic equation:
$s^2 + (1/RC)\ s + 1/LC = 0$

If the roots are:
Real and different or distinct (s1, s2): $v(t) = A_1e^{s1t} + A_2e^{s2t}$

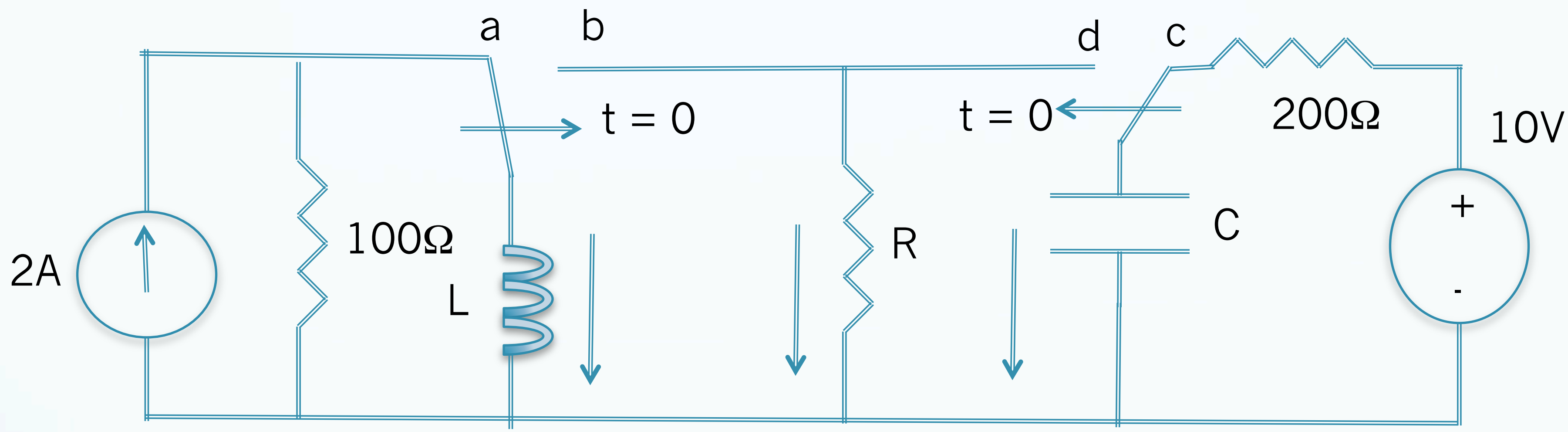
Real and equal (s1 = s2): $v(t) = D_1te^{-\alpha t} + D_2e^{-\alpha t}$

Complex (s1, s2): $v(t) = B_1e^{-\alpha t} \cos \omega_d t + B_2e^{-\alpha t} \sin \omega_d t$

where:
α = the neper frequency = 1/2RC
ω_d = the damping frequency = sqrt $(\omega_o{}^2 - \alpha^2)$
ω_o = the resonant frequency = 1/sqrt (LC)

DC Circuits: Transient Analysis

1. **Those circuits that contain resistors, inductors, and capacitors (RLC Circuits)**
2. **Those circuits responding to power removed from the circuit (Natural Response)**



Once we know the form of the solution by analyzing the roots of the characteristic equation, we can solve for the unknown coefficients in the differential equation via the initial conditions of the circuit.

For example, if the roots are real and distinct, and $v(t) = A_1e^{s1t} + A_2e^{s2t}$

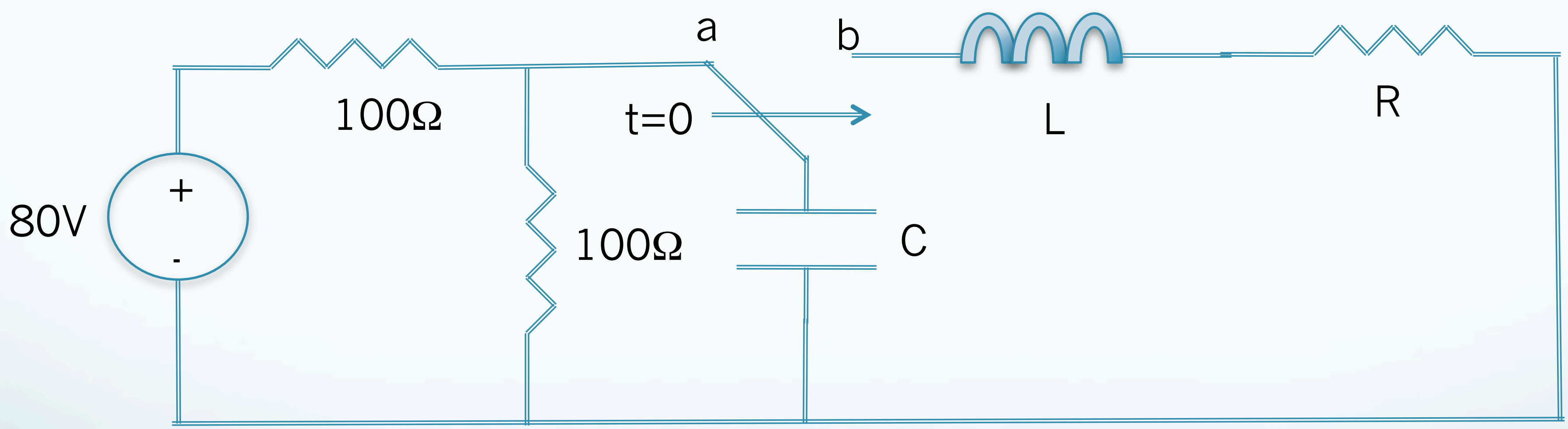
The initial voltage across the capacitor at t = 0: $10 = A_1 + A_2$

The initial current through the capacitor =
$-(10/R + 2) = C\, dv/dt = C\,[A_1s_1e^{s1t} + A_2s_2e^{s2t}] = C\,[A_1s_1 + A_2s_2]$

DC Circuits: Transient Analysis

In this lecture, we will look at the following circuits:

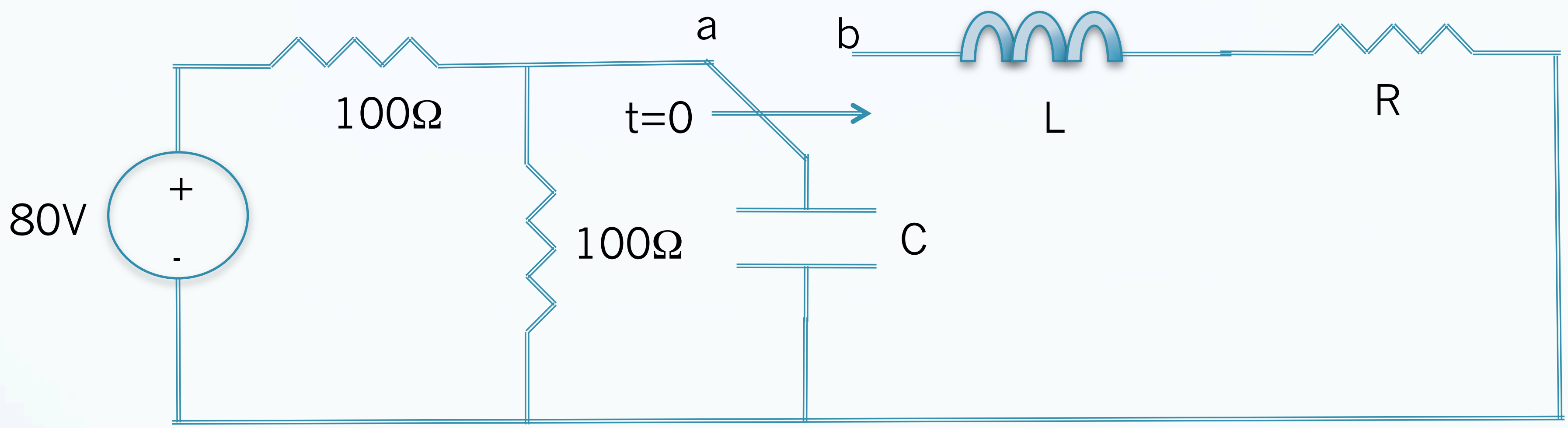
1. **Those circuits that contain resistors, inductors, and capacitors (RLC Circuits)**
2. **Those circuits responding to power removed from the circuit (Natural Response)**
3. Those circuits responding to power delivered/applied to the circuit (Step Response)



This circuit is an RLC circuit
which goes into its Natural Response at t > 0
Unlike the previous RLC circuit, however, this is a series RLC circuit
(rather than a parallel RLC circuit)

DC Circuits: Transient Analysis

1. **Those circuits that contain resistors, inductors, and capacitors (RLC Circuits)**
2. **Those circuits responding to power removed from the circuit (Natural Response)**

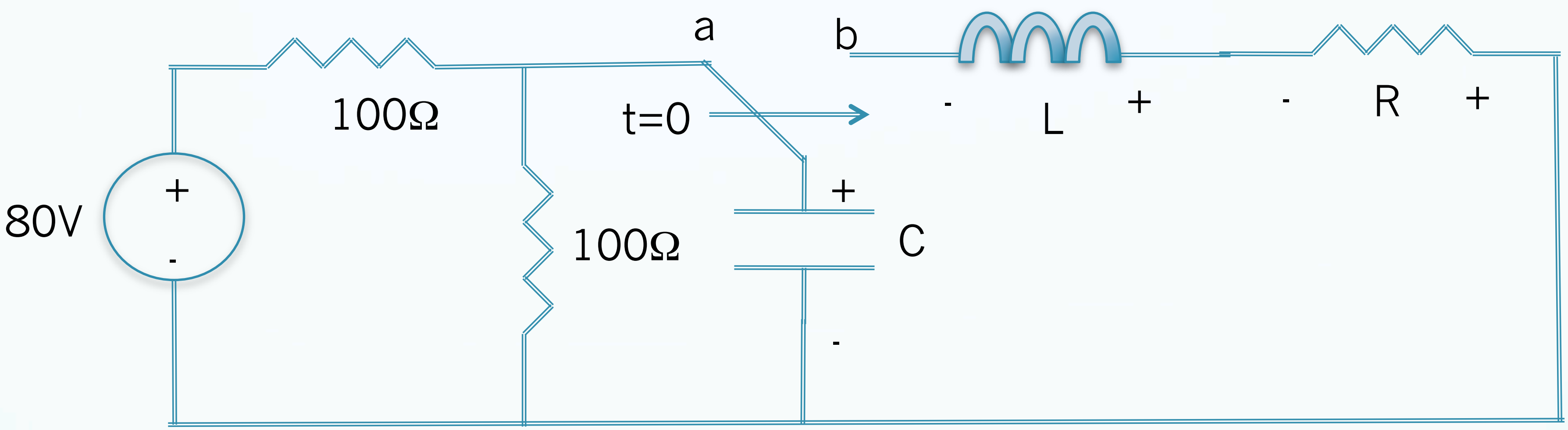


Right before t = 0 (at t = 0-):
The circuit is presumed to have had the switch in position A for "a long time" and is in DC Steady State; the inductor is acting like a short circuit and the capacitor like a short circuit.

	R	L	C
Voltage	0	0	40V
Current	0	0	0

DC Circuits: Transient Analysis

1. **Those circuits that contain resistors, inductors, and capacitors (RLC Circuits)**
2. **Those circuits responding to power removed from the circuit (Natural Response)**

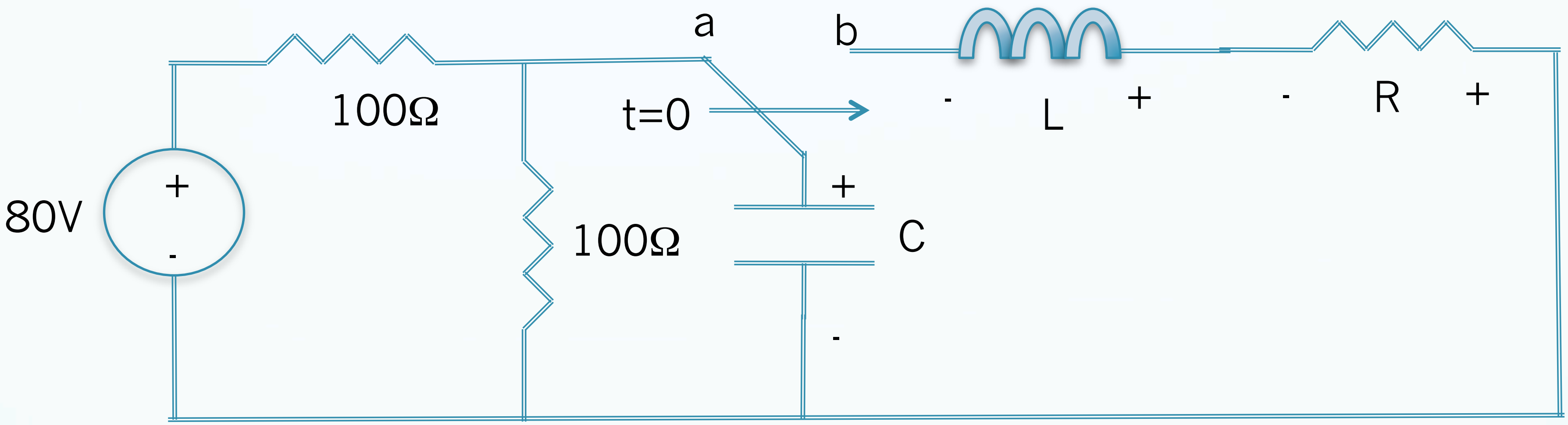


Right after t = 0 (at t = 0+):
Neither the capacitor voltage nor the inductor current can change instantaneously; we use this fact to calculate the current through and the voltage across the resistor R.

	R	L	C
Voltage	0	-40	40
Current	0	0	0

DC Circuits: Transient Analysis

1. **Those circuits that contain resistors, inductors, and capacitors (RLC Circuits)**
2. **Those circuits responding to power removed from the circuit (Natural Response)**

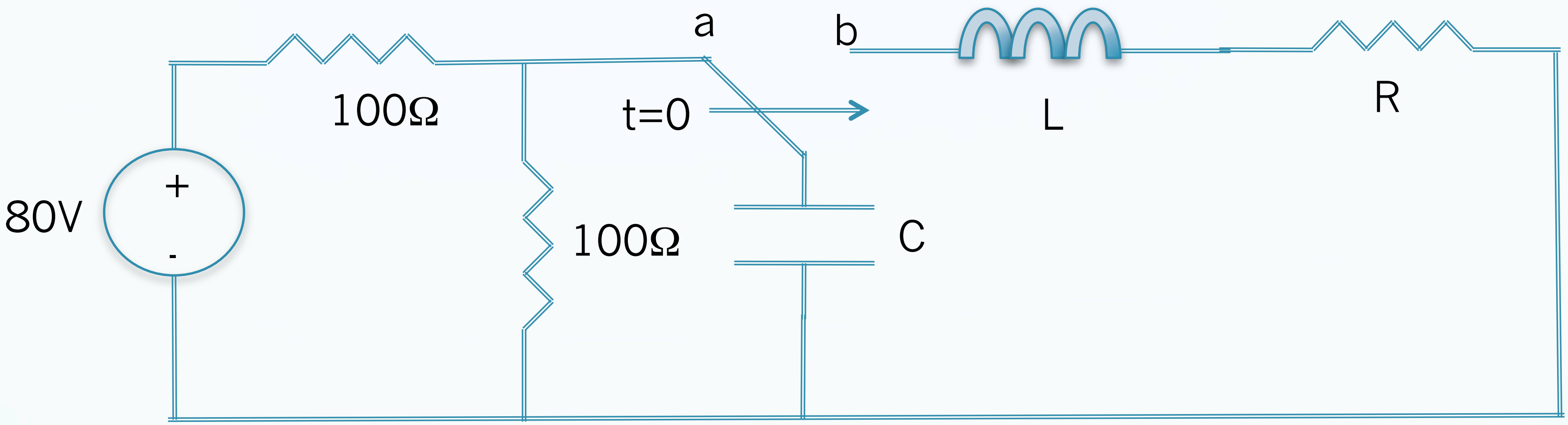


After the switch is in position b for a long time, and
The capacitor becomes an open circuit.
The inductor becomes a short circuit.

	R	L	C
Voltage	0	0	0
Current	0	0	0

DC Circuits: Transient Analysis

1. **Those circuits that contain resistors, inductors, and capacitors (RLC Circuits)**
2. **Those circuits responding to power removed from the circuit (Natural Response)**



At $t > 0$, KVL gives:
$V_L + V_R + V_C = 0$

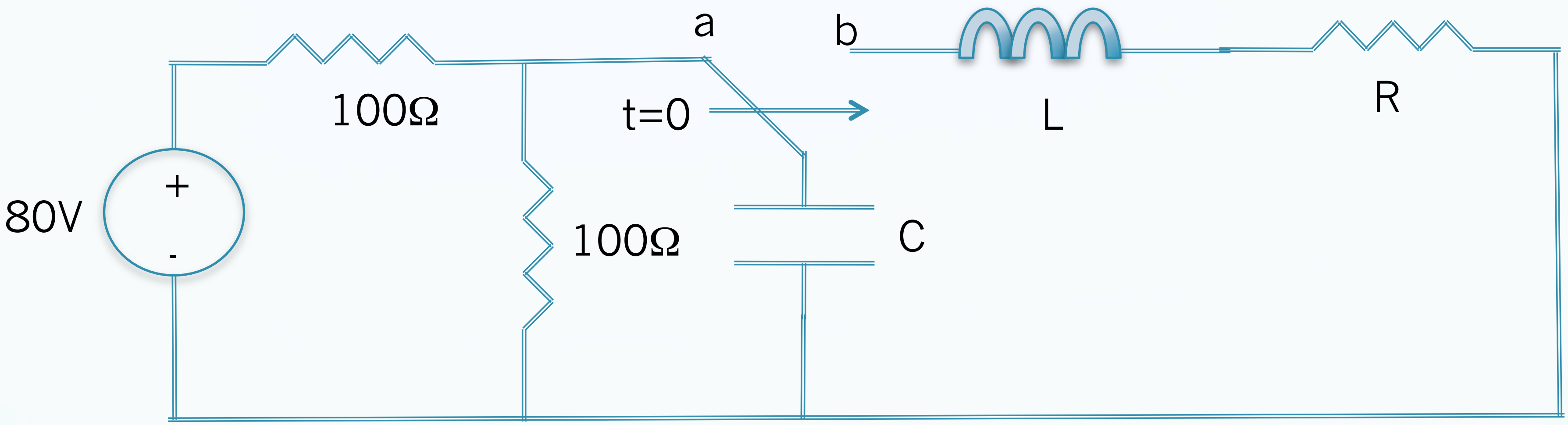
Substituting governing relationships for R, L, and C gives:

$$L\,di/dt + iR + 1/C \int i\,dt + V_o = 0$$

where i is the current through the inductor, capacitor, and resistor
Vo is the initial voltage across the capacitor.

DC Circuits: Transient Analysis

1. **Those circuits that contain resistors, inductors, and capacitors (RLC Circuits)**
2. **Those circuits responding to power removed from the circuit (Natural Response)**



Taking the derivative of the following expression:

$L\, di/dt + iR + 1/C \int i\, dt + V_o = 0$

gives a second order differential equation:

$(L)\, d^2i/dt^2 + R\, di/dt + (1/C)\, i = 0$

or:

$d^2i/dt^2 + (R/L)\, di/dt + (1/LC)\, i = 0$

DC Circuits: Transient Analysis

1. **Those circuits that contain resistors, inductors, and capacitors (RLC Circuits)**
2. **Those circuits responding to power removed from the circuit (Natural Response)**

Natural Response of the series RLC circuit:
Solving the second order differential equation:
$d^2i/dt^2 + (R/L)\, di/dt + (1/LC)\, i = 0$

Requires finding the roots of the characteristic equation:
$s^2 + (R/L)\, s + 1/LC = 0$

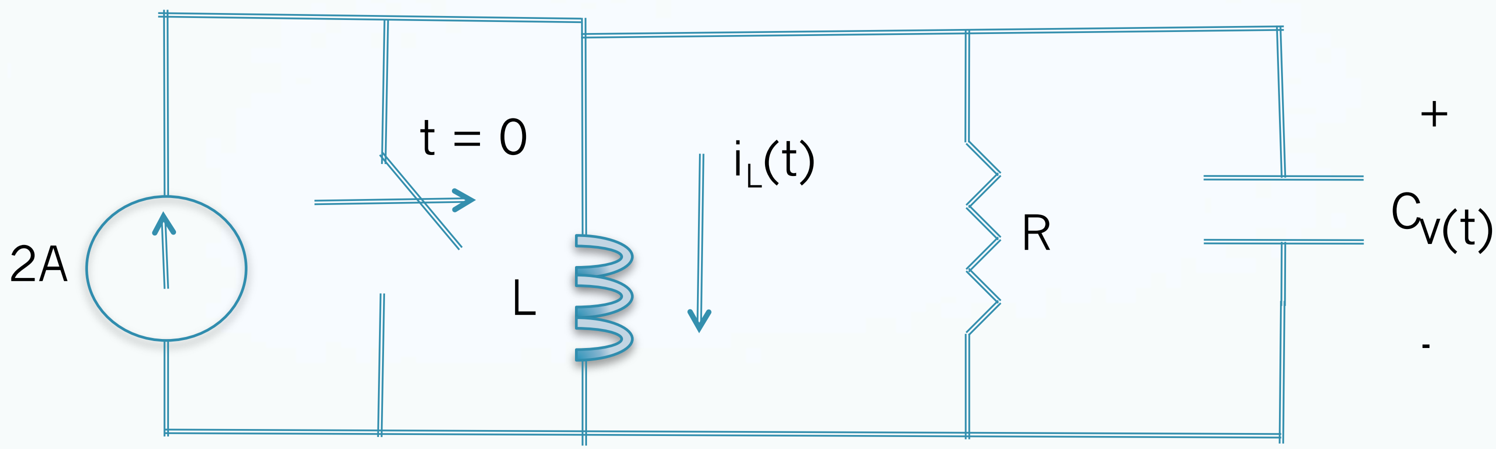
If the roots are:
Real and different or distinct (s1, s2): $i(t) = A_1 e^{s1t} + A_2 e^{s2t}$

Real and equal (s1 = s2): $i(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$

Complex (s1, s2): $i(t) = B_1 e^{-\alpha t} \cos \omega_d t + B_2 e^{-\alpha t} \sin \omega_d t$

where:
α = the neper frequency = R/2L
ω_d = the damping frequency = sqrt $(\omega_o^2 - \alpha^2)$
ω_o = the resonant frequency = 1/sqrt (LC)

DC RLC Circuits: Transient Analyses



At t>0, power is applied to the circuit. This circuit can be analyzed similarly to the natural response of the parallel RLC circuit. We can use an approach that makes the analysis very similar to that for the natural response.

First:

Find the form of the voltage v(t) as if the circuit were in a natural response, or
Find the form of the current $i_L(t)$ as if the circuit were in the natural response

Then:

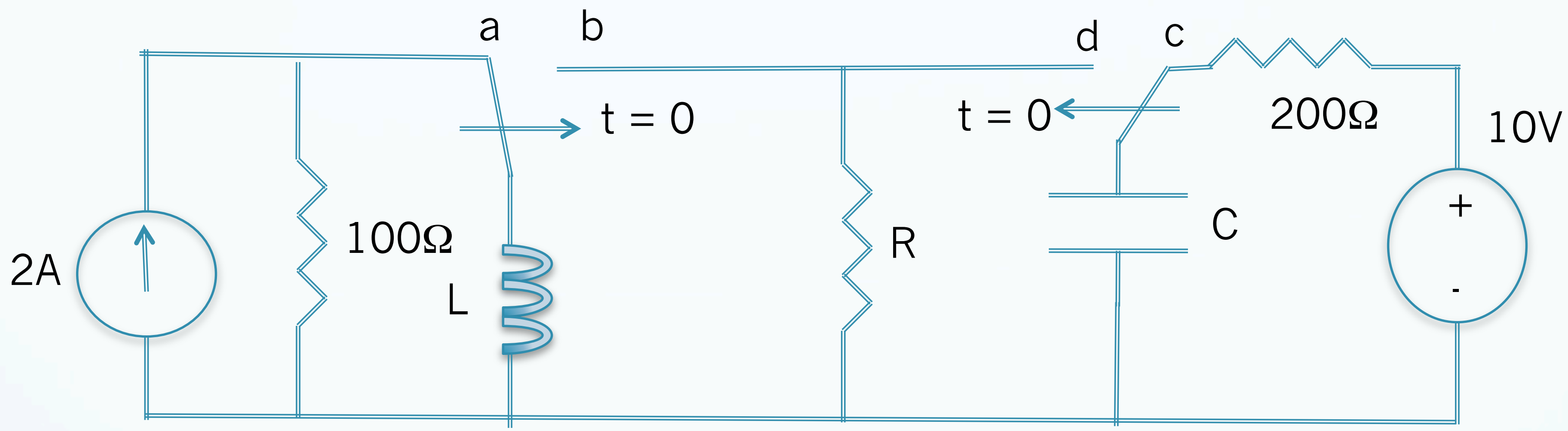
Find the initial conditions of the circuit:
v(t) and dv/dt at t = 0+ if solving for the voltage v(t)
i(t) and di/dt at t = 0+ if solving for the current i(t)

Then:

Solve for the coefficients in the form of the response using initial conditions and:
$v(t) = V_{final}$ + (form of natural response), or
$i(t) = I_{final}$ + (form of natural response)

DC RLC Circuits: Transient Analyses

Our discussion of the natural response of RLC circuits has included both parallel RLC (below) and series RLC circuits



Unlike RC and RL circuits, solving RLC circuits involves second order differential equations which can produce under-damped, over-damped, or critically damped responses. Simple KCL and KVL equations can be used to derive these second order differential equations. The solution of the characteristic equation (of the second order differential equation) yields the form of the response. Finally, the initial conditions on the inductor or capacitor yield the equations necessary to solve for the coefficients in the solution.