LECTURE 21
MODELS OF THE ATOM & SPECTRA

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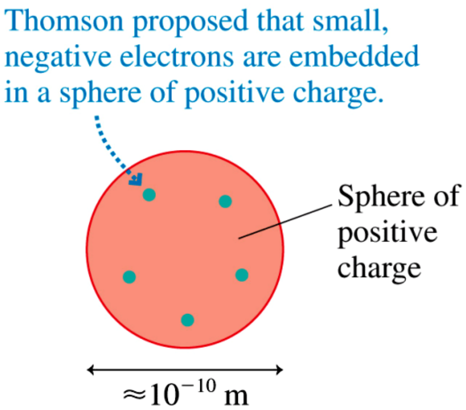
Lecture 21

- Reading chapter 31.1 to 31.4
  - Models of atoms
  - The spectrum of atomic hydrogen
  - Bohr’s model of the hydrogen atom
  - de Broglie waves and the Bohr model

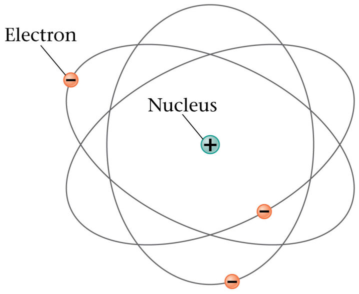
The plum-pudding model of the atom

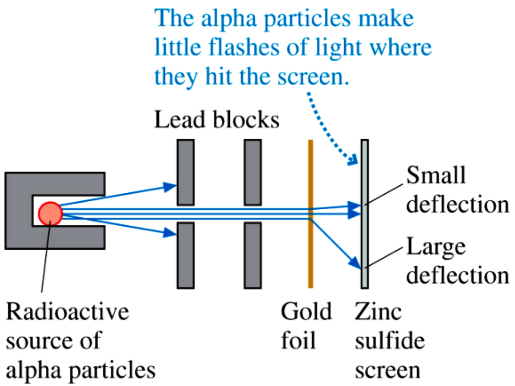
- In 1904, J. J. Thomson, who discovered electrons, proposed the "plum-pudding model."
  - It was known that electrons are much smaller and less massive than atoms.
  - Electrons are embedded like plums in a plum pudding.

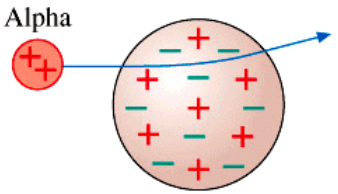


The miniature solar system model of the atom

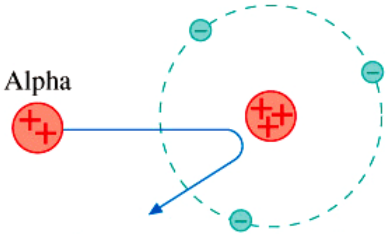
- Around 1911 Rutherford showed that the positive charge and most of the mass of an atom is concentrated in a very small region (~1 fm), now called the nucleus.
- The majority of alpha particles fired at gold atoms were only slightly deflected, but a small number were deflected by a large angle.







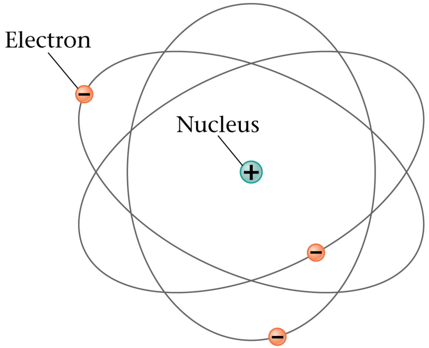
The alpha particle is only slightly deflected by a Thomson atom because forces from the spread-out positive and negative charges nearly cancel.



If the atom has a concentrated positive nucleus, some alpha particles will be able to come very close to the nucleus and thus feel a very strong repulsive force.

Quiz: 1

- Is there something wrong with the miniature solar system model of the atom? If so, what is wrong? Choose all that apply.
  A. No. Everything is fine with the model.
  B. Yes. Nothing is providing orbiting electrons the centripetal force.
  C. Yes. Orbiting electrons would collapse into the nucleus in a very short time.
  D. Yes. The frequency of radiation emitted by an electron would have a continuous spectrum.

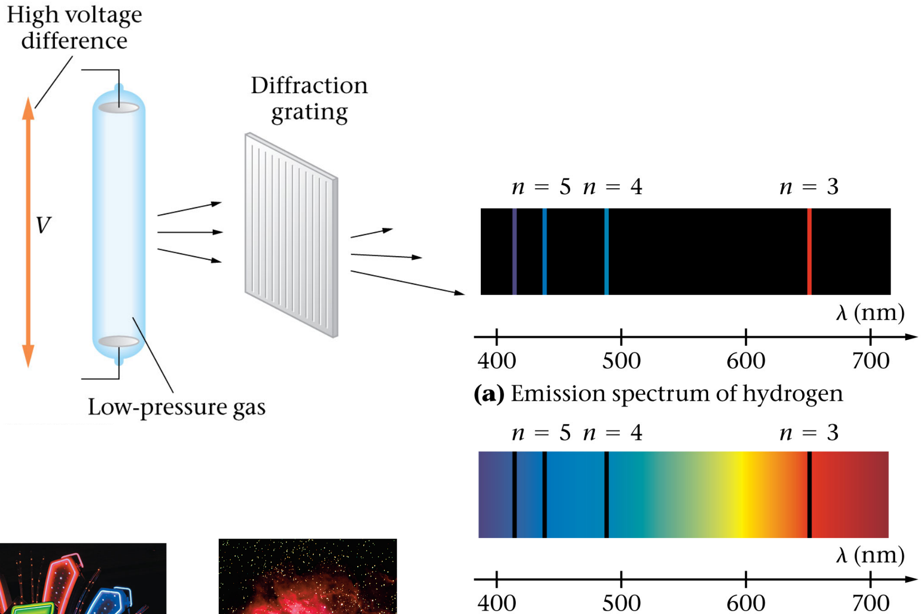


Quiz: 21-1 answer

- Yes. Orbiting electrons would collapse into the nucleus in a very short time.
- Yes. The frequency of radiation emitted by an electron would have a continuous spectrum.
- The coulomb force by the nucleus is providing orbiting electrons the centripetal force.
- Orbiting electrons are accelerating, and any accelerating charges should radiate and lose energy.
- The frequency of radiation emitted by a continuously radiating electron would have a continuous spectrum, rather than the individual frequencies that are actually observed.

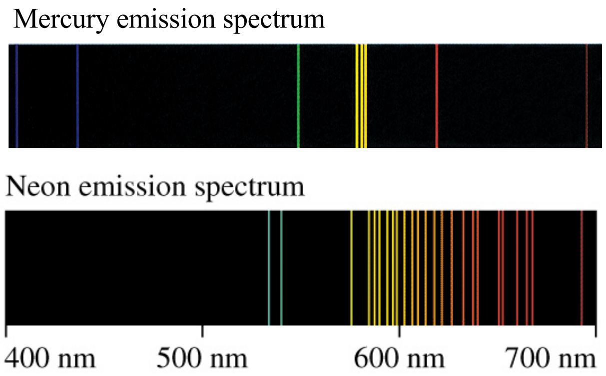
Atomic emission of light/Demo: 1

- Each atom has its own spectral lines.
- If white light passes through, it absorbs at the same frequencies seen in the emission spectrum.
- Demo: line spectra



(a) Emission spectrum of hydrogen

(b) Absorption spectrum of hydrogen



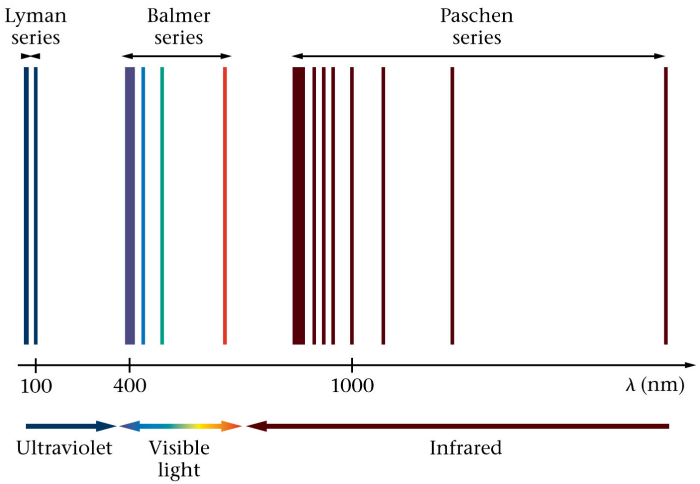
Atomic hydrogen spectrum

- The wavelengths in the spectrum of hydrogen are given by:

$$\boxed{\frac{1}{\lambda} = R\left(\frac{1}{n'^2} - \frac{1}{n^2}\right)}$$

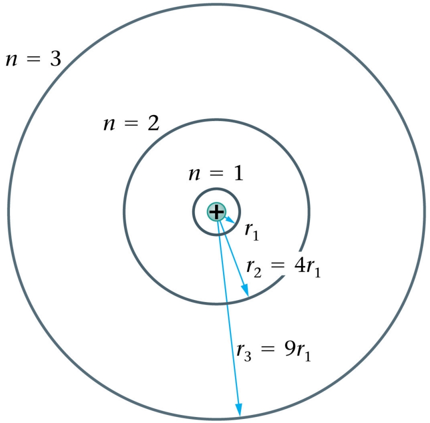
  - $n' = 1, 2, 3, \cdots$
  - $n = n' + 1, n' + 2, n' + 3, \cdots$
  - $R = 1.097\times10^{7}\ \mathrm{m}^{-1}$ is the Rydberg constant.

n'	Series name
1	Lyman
2	Balmer
3	Paschen
4	Brackett
5	Pfund



Bohr's model of the hydrogen-like atom

- Bohr's model gives correct predictions for any single-electron atom with the following assumptions:
  - The electrons moves in a circular orbit around the nucleus.
  - Only certain orbits are allowed. Allowed orbits must have an angular momentum equal to $L_n = \frac{nh}{2\pi}, n = 1, 2, 3, \cdots$.
  - Electrons in allowed orbits do not radiate.
  - When an electron changes from one orbit to another, radiation with a frequency $|\Delta E| = hf$ is emitted or absorbed.

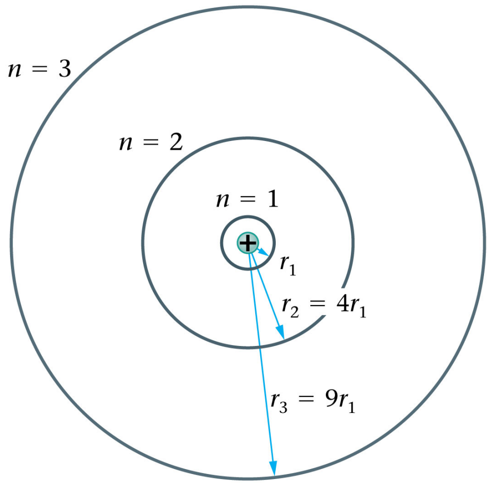


Orbits in Bohr's model of the hydrogen-like atom

- The electrons moves in a circular orbit with a radius

$$\boxed{r_n = \left(\frac{h^2}{4\pi^2 mkZe^2}\right) n^2 = \frac{r_1}{Z} n^2}$$

  - $n = 1, 2, 3, \cdots$
  - $r_1 = 0.0529\ \mathrm{nm}$ is the Bohr radius.



Energies in Bohr's model of the hydrogen-like atom

- There is an energy level in the hydrogen-like atom corresponding to each allowed state radius.

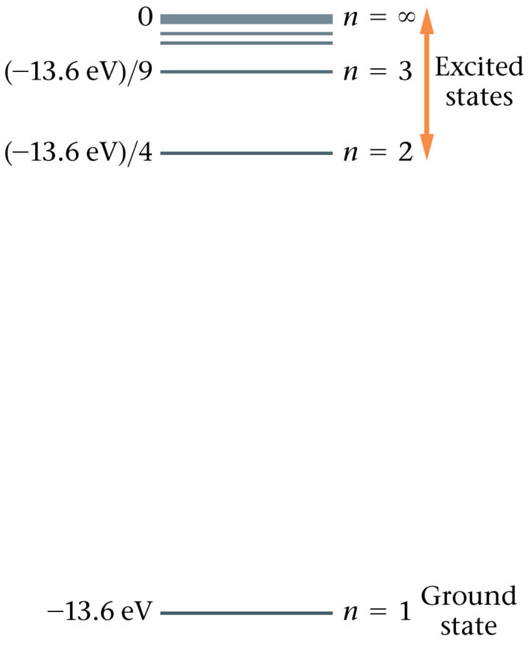
$$\boxed{E_n = (-13.6\ \mathrm{eV})\frac{Z^2}{n^2}}$$

  - $n = 1, 2, 3, \cdots$

- The radiation emitted from excited Bohr's atoms as they de-excite agrees with the observation:

$$\frac{1}{\lambda} = \left(\frac{2\pi^2 m k^2 e^4}{h^3 c}\right)\left(\frac{1}{n_\mathrm{f}^2} - \frac{1}{n_\mathrm{i}^2}\right)$$

Energy-level diagram

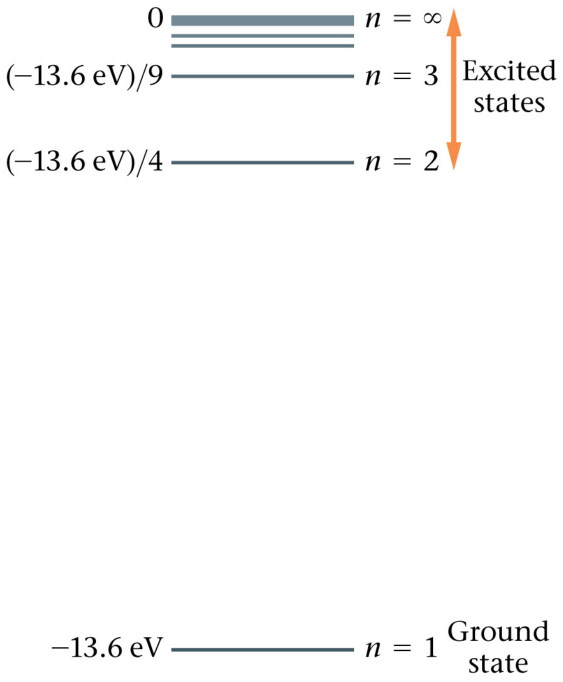


Quiz: 2

- How much energy in eV does it take to ionize a hydrogen atom in its ground state?

Quiz: 21-2 answer

- 13.6 eV
- When a hydrogen atom in the ground state absorbs 13.6 eV of energy, the electron makes a transition to $n = \infty$ state, which means that the electron is no longer associated to the nucleus.



Quiz: 3

- How much energy in eV does it take to change a He^+ ion in its ground state into a He^{++} ion in its ground state? Keep in mind that the nucleus of a helium atom has two protons.

Quiz: 21-3 answer

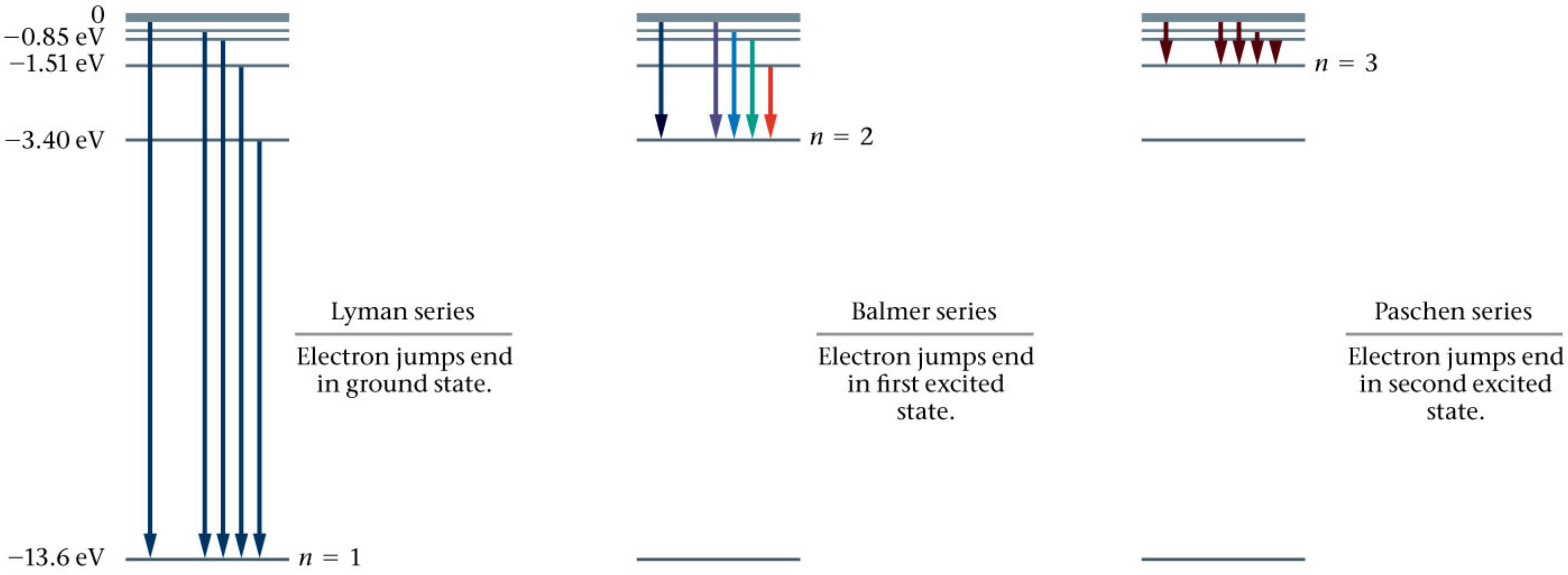
- 54.4 eV
- He^+ ion in its ground state has an energy of $E_n = (-13.6\text{ eV})\frac{Z^2}{n^2} = (-13.6\text{ eV})\frac{2^2}{1^2} = -54.4\text{ eV}$
- So, you need to add 54.4 eV of energy to the electron for it to make a transition to $n = \infty$ state, which means that the electron is no longer associated to the nucleus, leaving behind a He^{++} ion in its ground state.

Quiz: 4

- When a broad spectrum of light passes through hydrogen gas at room temperature, absorption lines are observed that correspond only to the Balmer ($n_{\mathrm{f}} = 2$) series. Why aren't other series observed?
  A. They're there, but they're invisible.
  B. Only the Balmer series can be excited at room temperature.
  C. The other series have been ionized.
  D. All the photons have been used up.

Quiz: 21-4 answer

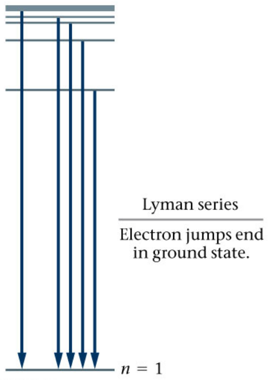
- They're there, but they're invisible.
- Most atoms at room temperature are in the ground state.
- The Balmer series is the only one that involves wavelengths in the visible part of the spectrum!



Example 1

a) What is the wavelength of the least energetic photon emitted in the Lyman series of the hydrogen atom spectrum lines?

b) What is the wavelength of the most energetic photon emitted in the Lyman series?

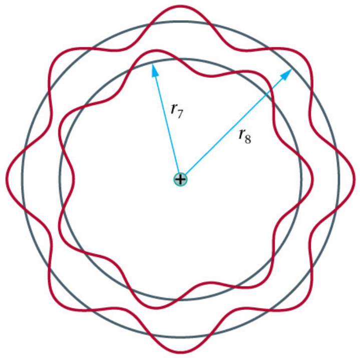


Quiz: 5

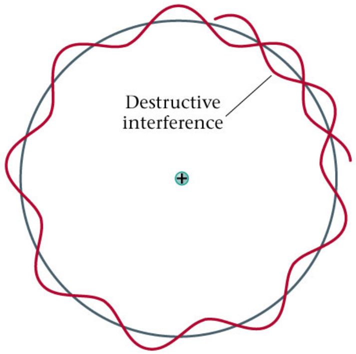
- De Broglie showed that the allowed orbits of the Bohr model correspond to standing matter wave of the electrons. What is the relationship between the wavelengths of the electrons in an atom and the circumferences of the allowed orbits?
  A. The circumference must be an integer multiple of λ.
  B. The circumference must be an odd integer multiple of $\frac{\lambda}{2}$.
  C. There is not relationship between the circumference and λ.

Quiz: 21-5 answer

- The circumference must be an integer multiple of λ.
- The circumference of an allowed orbit is $2\pi r = n\lambda, n = 1, 2, 3, \cdots$.
- The wavelength of the electron is $\lambda = \dfrac{h}{p} = \dfrac{2\pi r}{n}$.
- The angular momentum is $L = rp = \dfrac{nh}{2\pi}$ in agreement with Bohr's assumption.



(a)



(b)

Schrödinger equation for an electron in an atom

- In the Schrödinger theory of quantum mechanics, the electron is described by a wave function, which gives the probability of finding the electron at a given position and time.
- The Schrödinger's equation you absolutely ***DO NOT*** need to know is given by

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + U(\vec{\mathbf{r}},t)\right]\Psi(\vec{\mathbf{r}},t) = i\hbar\frac{\partial}{\partial t}\Psi(\vec{\mathbf{r}},t)$$

- In plain language, it means "total energy equals kinetic energy plus potential energy."

Quantum weirdness

- Bohr used to say "if you aren't confused by quantum mechanics, then you don't really understand it."
- Based on the QM of a two-particle system, Einstein, Podolsky, and Rosen predicted results of a thought-experiment that were too weird for Einstein to accept QM.
- Schrödinger's cat
  - Suppose a cat is placed in a sealed box containing a device that has a 50% chance of killing the cat.
  - The cat is dead AND alive until you open the box and check the status of the cat.
  - When you make the observation, the cat is either dead OR alive.